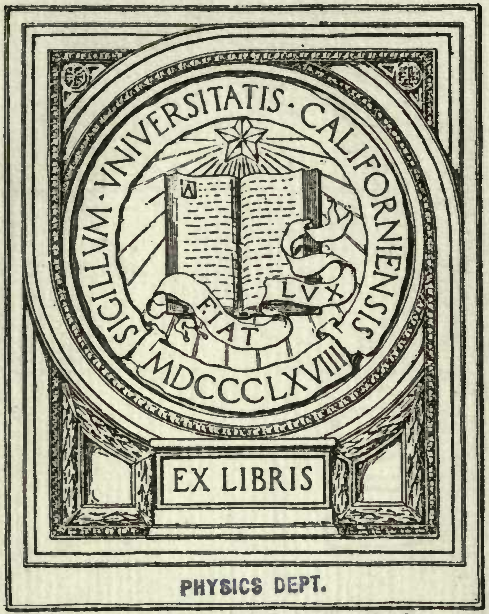

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SOME CONTRIBUTIONS

FROM THE

LABORATORY OF PHYSICS

OF THE

UNIVERSITY OF ILLINOIS

URBANA, ILLINOIS

for 1914 - 1919
In Two Parts

PART I

PHYSICS DEPT.

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PHYSICS DEPT.

SOME CONTRIBUTIONS FROM THE LABORATORY OF PHYSICS, UNIVERSITY OF ILLINOIS

for 1914-1919

Part I

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN NO. 73 MARCH, 1914

ACOUSTICS OF AUDITORIUMS.

BY F. R. WATSON, ASSISTANT PROFESSOR OF PHYSICS.

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UNIV. OF CALIFORNIA

Fig. 1. Auditorium, University of Illinois.

ACOUSTICS OF AUDITORIUMS

An Investigation of the Acoustical Properties of the Auditorium at the University of Illinois.

I. Introduction.

Much concern has arisen in late years in the minds of architects because of the faulty acoustics that exist in many auditoriums. The prevalence of echoes and reverberations with the consequent difficulty in hearing and understanding on the part of the auditor defeats the purpose of the auditorium and diminishes its value.

The Auditorium at the University of Illinois presents such a case. The building is shaped nearly like a hemisphere, with several large arches and recesses to break up the regularity of its inner surface. The original plans of the architect were curtailed because of insufficient money appropriated for the construction. The interior of the hall, therefore, was built absolutely plain with almost no breaking up of the large, smooth wall surfaces; and, at first, there were no furnishings except the seats and the cocoa matting in the aisles. The acoustical properties proved to be very unsatisfactory. A reverberation or undue prolongation of the sound existed, and in addition, because of the large size of the room and the form and position of the walls, echoes were set up.

If an observer stood on the platform and clapped his hands, a veritable chaos of sound resulted. Echoes were heard from every direction and reverberations continued for a number of seconds before all was still again. Speakers found their utterances thrown back at them, and auditors all over the house experienced difficulty in understanding what was said. On one occasion the University band played a piece which featured a xylophone solo with accompaniment by the other instruments. It so happened that the leader heard the echo more strongly than the direct sound and beat time with it. Players near the xylophone kept time to the direct sound, while those farther away followed the echo. The confusion may well be imagined.

Thus it seemed that the Auditorium was doomed to be an acoustical horror; that speakers and singers would avoid it, and that auditors would attend entertainments in it only under protest. But the apparent misfortune was in one way a benefit since it provided an opportunity to

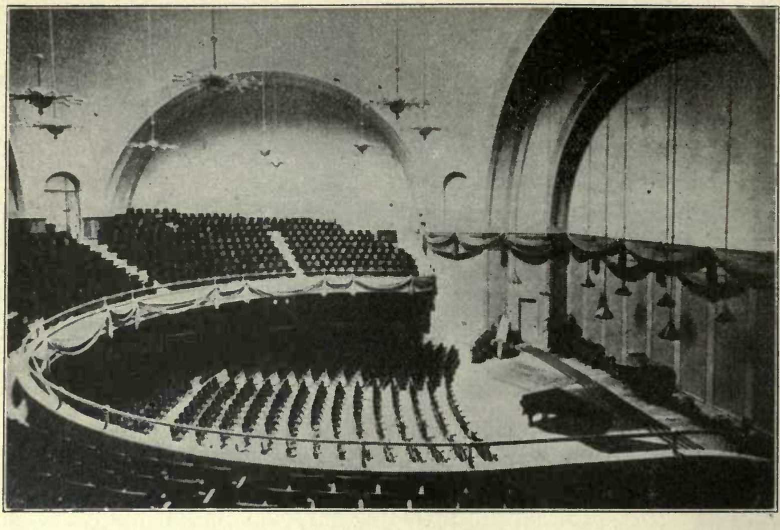

FIG. 2. PHOTOGRAPH OF INTERIOR. VIEW OF STAGE.

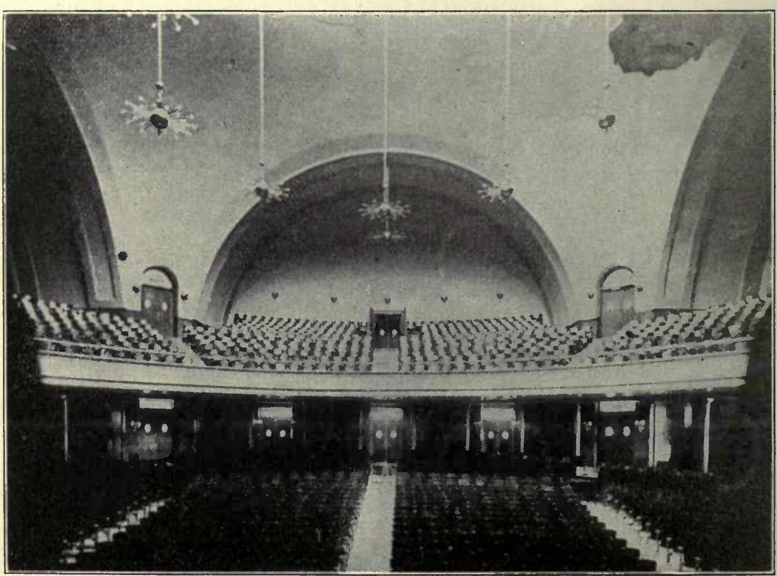

FIG. 3. PHOTOGRAPH OF INTERIOR. VIEW TOWARD BALCONY.

study defective acoustics under exceptionally good conditions and led to conclusions that not only allowed the Auditorium to be improved but also indicate some of the pitfalls to be avoided in future construction of other halls.

An investigation of the acoustical properties of the Auditorium was begun in 1908 and has continued for six years. It was decided at the outset not to use "cut and try" methods of cure, but to attack the problem systematically so that general principles could be found, if possible, that would apply not only to the case being investigated but to auditoriums in general. This plan of procedure delayed the solution of the problem, since it became necessary to study the theory of sound and carry out laboratory investigations at the same time that the complex conditions in the Auditorium were being considered. The author spent one year of the six abroad studying the theory of acoustics and inspecting various auditoriums.

The main echoes in the Auditorium were located by means of a new method for tracing the path of sound, the time of reverberation was determined by Sabine's method, and a general diagnosis of the acoustical defects was made. Hangings and curtains were installed in accordance with the results of the study so that finally the acoustical properties were improved.

Acknowledgment.—The author desires to express his great appreciation of the advice and encouragement given by President E. J. James, Supervising Architect J. M. White, and Professor A. P. Carman of the Physics Department. He desires also to acknowledge the material assistance cheerfully rendered by the workmen at the University, which contributed in no small degree to the successful solution of the problem.

II. Behavior of Sound Waves in a Room.

When a speaker addresses an audience, the sounds he utters proceed in ever widening spherical waves until they strike the boundaries of the room. Here the sound is partly reflected, partly transmitted, and the rest absorbed. The amounts of reflection, absorption and transmission depend on the character of the walls. A hard, smooth wall reflects most of the sound so that but little is transmitted or absorbed. In the case of a porous wall or a yielding wall, the absorption and transmission are greater, and the reflection is less. After striking a number of reflecting surfaces, the energy is used up and the sound dies out.

The reflection of sound produces certain advantages and disadvantages for the acoustics. When it is considered that sound travels about 1100 feet a second it may be seen that a room of ordinary size is almost immediately filled with sound because of the many reflections. In a room 40 feet square, for instance, the number of reflections per second between opposite walls is 1100 ÷ 40, or approximately 27. The number is really greater than this, since the sound that goes into the corners is reflected much more frequently than out in the middle where the distances between walls are greater. The result is that the sound mixes thoroughly in all parts of the room so as to give the same average intensity; that is, the sound is of the same average *loudness* for all auditors, even for those in the remotest corners.

Though the reflection of sound has the advantage of fulfilling the conditions for loudness, it introduces at the same time possibilities for setting up defective acoustics. For instance, when the walls of the room are hard and smooth very little energy is lost at each impact of the sound and many reflections take place before it finally dies out. This slow decadence of the sound, or *reverberation* as it is called, is the most common defect in auditoriums.

If a speaker talks in such a hall the auditors have difficulty in understanding. Each sound, instead of dying out quickly, persists for some time so that the succeeding words blend with their predecessors and set up a mixture of sounds which produces confusion. The cure for the trouble is brought about by the introduction of materials such as carpets, tapestries, and the like, which act as absorbers of sound and reduce the time of reverberation.

When music is played in an auditorium with a prolonged reverberation, the tones following one another blend and produce the same effect as that of a piano when played with the loud pedal in use. A reverberation is more advantageous for music than for speech, since the prolongation and blending of the musical tones is desired, but the mixing of the words in a speech is a distinct disadvantage. When curing this defect for halls used for both music and speaking, a middle course must be steered, so that the reverberation is made somewhat long for speaking and somewhat short for music, yet fairly satisfactory for both.

Going back to the consideration of the reflection of sound, it is found that another defect may be produced, namely, an *echo*. This is the case when a wall at some distance reflects the sound to the position of the auditor. He hears the sound first from the speaker, then later by reflec-

tion from the wall. The time interval between the direct and reflected sound must be great enough to allow two distinct impressions to be made. This time is about 1/15 of a second, but varies with the acuteness of the observer. The farther off the wall is, the greater is the time interval and the more pronounced is the echo. If the wall is not very distant, the time interval is too short to allow two distinct impressions to be made, and the effect on the auditor is then much the same as if his neighbor at his side speaks the words of the discourse in his ear at the same time that he gets them directly from the speaker. In case the reflecting wall is curved so as to focus the sound the echoes are much more pronounced. A curved wall wherever it may be placed in an auditorium is thus always a menace to good acoustics.

There are other actions of the sound that may result in acoustical defects. The phenomena of *resonance,* for instance, may cause trouble. Suppose that the waves of sound impinge on an elastic wall, not too rigid. If these waves are timed right they set the wall in vibration in the same way that the bell ringer causes a bell to ring by a succession of properly timed pulls on the bell rope. The wall of the room will then vibrate under the action of the sound with which it is in tune and will reinforce it. Now suppose a band is playing in a room. Certain tones are reinforced, while the others are not affected. The original sound is then distorted. The action is the same on the voice of the speaker. The sounds he utters are complex and as they reach the walls certain components are reinforced and the quality of the sound is changed. This action of resonance may also be caused by the air in a room. Each room has a definite pitch to which it responds, the smaller the volume of the room the higher being the pitch. A large auditorium would respond to the very low pitch of the bass drum. In small rooms and alcoves the response is made to higher pitched tones, as may be observed by singing the different notes of the scale until a resonance is obtained.

Another action of sound causes the *interference* of waves. Thus the reflected waves may meet the oncoming ones and set up concentrations of sound in certain positions and a dearth of sound in others.

Summing up, it is seen that the effects of sound which may exist in a room are *loudness, reverberation, echoes, resonance,* and *interference,* and that the most common defects are reverberation and echoes. We now turn to the discussion of the methods of cure.

III. Methods of Improving Faulty Acoustics.

A. Reverberation and its Cure.

Everyone has doubtless observed that the hollow reverberations in an empty house disappear when the house is furnished. So, in an auditorium, the reverberation is lessened when curtains, tapestries, and the like are installed in sufficient numbers. The reason for this action is found when we inquire what ultimately becomes of the sound.

Sound is a form of energy and energy can not be destroyed. When it finally dies out, the sound must be changed to some other form of energy. In the case of the walls of a room, for instance, it has been shown in a preceding paragraph that the sound may be changed into mechanical energy in setting these walls in vibration. Again, some of the sound may pass out through open windows and thus disappear. The rest of the sound, according to Lord Rayleigh, is transformed by friction into heat. Thus[1] a high pitched sound, such as a hiss, before it travels any great distance is killed out by the friction of the air. Lower pitched sounds, on reaching a wall, set up a friction in the process of reflection between the air particles and the wall so that some of the energy is converted into heat.[2] The amount of sound energy thus lost is small if the walls are hard and smooth. The case is much different, however, if the walls are rough and porous, since it appears that the friction in the pores dissipates the sound energy into heat. In this connection, Lamb[3] writes: "In a sufficiently narrow tube the waves are rapidly stifled, the mechanical energy lost being of course converted into heat. * * * * When a sound wave impinges on a slab which is permeated by a large number of very minute channels, part of the energy is lost, so far as the sound is concerned, by dissipation within these channels in the way just explained. The interstices in hangings and carpets act in a similar manner, and it is to this cause that the effect of such appliances in deadening echoes in a room is to be ascribed, a certain proportion of the energy being lost at each reflection. It is to be observed that it is only through the action of true dissipative forces, such as viscosity and thermal conduction, that sound can die out in an enclosed space, no mere modifications of the waves by irregularities being of any avail."

It should be pointed out in this connection that any mechanical breaking up of the sound by relief work on the walls or by obstacles in the room will not primarily diminish the energy of the sound. These

1. "Theory of Sound," Vol. II, p. 316.
2. "Theory of Sound," Vol. II, § 351.
3. "Dynamical Theory of Sound," p. 196.

may break up the regular reflection and eliminate echoes, but the sound energy as such disappears only when friction is set up.

The following quotation from Rayleigh[1] emphasizes these conclusions: "In large spaces, bounded by non-porous walls, roof, and floor, and with few windows, a prolonged resonance seems inevitable. The mitigating influence of thick carpets in such cases is well known. The application of similar material to the walls and roof appears to offer the best chance of further improvement."

Experimental Work on Cure of Reverberation.—The most important experimental work in applying this principle of the absorbing power of carpets, curtains, etc., has been done by Professor Wallace C. Sabine of Harvard University.[2] In a set of interesting experiments lasting over a period of four years, he was able to deduce a general relation between t, the time of reverberation, V, the volume of the room, and a, the absorbing power of the different materials present. Thus:

$$t = 0.164 \ V \div a \qquad (1)$$

For good acoustical conditions, that is, for a short time of reverberation, the volume V should be small and the absorbing materials, represented by a, large. This is the case in a small room with plenty of curtains and rugs and furniture. If, however, the volume of the room is great, as in the case of an auditorium, and the amount of absorbing materials small, a troublesome reverberation will result.

Professor Sabine determined the absorbing powers of a number of different materials. Calling an open window a perfect absorber of sound, the results obtained may be written approximately as follows:

One square meter of open window space	1.000
One square meter of glass, plaster, or brick	.025
One square meter of heavy rugs, curtains, etc.	.25
One square meter of hair felt, 1 inch thick	.75
One square meter of audience	.96

These values, together with the formula, allow a calculation to be made in advance of construction for the time of reverberation. This pioneer work cleared the subject of architectural acoustics from the fog of mystery that hung over it and allowed the essential principles to be seen in the light of scientific experiment.

In a later investigation[3] Sabine showed that the reverberation depended also on the pitch of sound. As a concrete example, the high

1. "Theory of Sound," p. 333.
2. "Architectural Acoustics." A series of articles in the Engineering Record, 1900; also the American Architect, 1900.
3. "Architectural Acoustics," Proc. of Amer. Acad. of Arts and Sciences. Vol. 42, pp. 49-84, 1906.

notes of a violin might be less reverberant with a large audience than the lower tones of the bass viol, although both might have the same reverberation in the room with no audience. Again, the voice of a man with notes of low pitch might give satisfactory results in an auditorium while the voice of a woman with higher pitched notes would be unsatisfactory.

These considerations show that the acoustics in an auditorium vary with other factors than the volume of the room and the amount of absorbing material present. The audience may be large or small, the speaker's voice high or low, the entertainment a musical number or an address. The best arrangement for good acoustics is then a compromise where the average conditions are satisfied. The solution offered by Professor Sabine is such an average one, and has proved satisfactory in practice.

The problem of architectural acoustics has been attacked experimentally by other workers. Stewart[1] proposed a cure for the poor acoustical conditions in the Sibley Auditorium at Cornell University. His experiments confirmed the work of Sabine. Marage[2], after investigating the properties of six halls in Paris, approved Sabine's results and advocated a time of reverberation of from ½ to 1 second for the case of speech.

Formulae for Reverberation of Sound in a Room.—On the theoretical side, Sabine's formula has been developed by Franklin[3], who obtained the relation $t = 0.1625\ V \div a$, an interesting confirmation, since Sabine's experimental value for the constant was 0.164.

A later development has been given by Jäger,[4] who assumes for a room whose dimensions are not greater than about 60 feet, that the sound, after filling the room, passes equally in all directions through any point, and that the average energy is the same in different parts of the room. By using the theory of probability and considering that a beam of sound in any direction may be likened to a particle with a definite velocity, he was able to deduce Sabine's formula and write down the factors that enter into the constants. Applying his results to the case of reflection of sound from a wall, he showed that sound would be reflected in greater volume when the mass of the wall was increased and

1. G. W. Stewart. "Architectural Acoustics," Sibley Journal of Engineering, May, 1903. Published by Cornell University, Ithaca, N. Y.

2. "Qualités acoustiques de certaines salles pour la voix parlée." Comptes Rendus, 142, 878, 1906.

3. W. S. Franklin. "Derivation of Equation of Decaying Sound in a Room and Definition of Open Window Equivalent of Absorbing Power." Physical Review, Vol. 16, pp. 372-374, 1903.

4. G. Jäger. "Zur Theorie des Nachhalls." Sitzungsberichte der Kaiserliche Akad. der Wissenschaften in Wien, Math-naturw. Klasse; Bd. CXX. Abt. 11 a. Mai, 1911.

the pitch of the sound made higher. He showed also that when sound impinges on a porous wall, more energy is absorbed when the pitch of the sound is high than when it is low, since the vibrations of the air are more frequent, and more friction is introduced in the interstices of the material.

B. ECHOES AND THEIR REMEDY.

An echo is set up by a reflecting wall. If an observer stands some distance from the front of a cliff and claps his hands, or shouts, he finds that the sound is returned to him from the cliff as an echo. So, in an auditorium, an auditor near the speaker gets the sound first directly from the speaker, then, an instant later, a strong repetition of the sound by reflection from a distant wall. This echo is more pronounced if the wall is curved and the auditor is at the point where the sound is focused.

To cure such an echo, two methods may be considered. One method consists in changing the form of the wall so that the reflected sound no longer sets up the echo. That is, either change the angle of the wall, so that the reflected sound is sent in a new direction where it may be absorbed or where it may reinforce the direct sound without producing any echoes, or else modify the surface of the wall by relief work or by panels of absorbing material, so that the strong reflected wave is broken up and the sound is scattered. The second method is to make the reflecting wall a "perfect" absorber, so that the incident sound is swallowed up and little or none reflected. These methods have been designated as "surgical" and "medicinal" respectively. Each method has its disadvantages. Changing the form of the walls in an auditorium is likely to do violence to the architectural design. On the other hand, there are no perfect absorbers, except open windows, and these can seldom be applied. The cure in each case is, then, a matter of study of the special conditions of the auditorium. Usually a combination of the surgical and the medicinal cures is adopted. For instance, coffering a wall so that panels of absorbing material may be introduced has been found to work well in bettering the acoustics, and also, in many cases, it fits in with the architectural features.

C. POPULAR CONCEPTION OF CURES.—USE OF WIRES AND SOUNDING BOARDS.

A few words should be written concerning the popular notion that wires and sounding boards are effective in curing faulty acoustics. Experiments and observations show that wires are of practically no benefit,

and sounding boards can be used only in special cases. Wires stretched in a room scarcely affect the sound, since they present too small a surface to disturb the waves. They have much the same effect on sound waves that a fish line in the water has on water waves. The idea has, perhaps, grown into prominence because of the action of a piano in responding to the notes of a singer. The piano has every advantage over a wire in an auditorium. It has a large number of strings tuned to different pitches so that it responds to any note sung. It also has a sounding board that reinforces strongly the sound of the strings. Finally, the singer is usually near the piano. The wire in the auditorium responds to only one tone of the many likely to be present, it has no sounding board, and the singer is some distance away. But little effect, therefore, is to be expected.

The author has visited a number of halls where wires have been installed, and has yet to find a case where pronounced improvement has resulted.[1] Sabine[2] cites a case where five miles of wire were stretched in a hall without helping the acoustical conditions. It is curious that so erroneous a conception has grown up in the public mind with so little experimental basis to support it.

Sounding Boards.—Sounding boards or, more properly, reflecting boards, have value in special cases. Some experiments are described later where pronounced effects were obtained. The sounding board should be of special design to fit the conditions under which it is to be used.

Modeling New Auditoriums after Old Ones with Good Acoustics.—Another suggestion often made is for architects to model auditoriums after those already built that have good acoustical properties. It does not follow that halls so modeled will be successful, since the materials used in construction are not the same year after year. For instance, a few years ago it was the usual custom to put lime plaster on wooden lath; now it is frequently the practice to put gypsum plaster on metal lath, which forms an entirely different kind of a surface. This latter arrangement makes hard, non-porous walls which absorb but little sound, and thus aggravate the reverberation. Further, a new hall usually is changed somewhat in form from the old one, to suit the ideas of the architect, and it is very likely that the changes will affect the acoustics.

1. Science, Vol. 35, p. 833, 1912.
2. Arch. Quarterly of Harvard University, March, 1912.

D. THE EFFECT OF THE VENTILATION SYSTEM ON THE ACOUSTICS.

At first thought it might seem that the ventilation system in a room would affect the acoustical properties. The air is the medium that transmits the sound. It has been shown that the wind has an action in changing the direction of propagation of sound.[1] Sound is also reflected and refracted at the boundary of gases that differ in density and temperature.[2] It is found, however, that the effect of the usual ventilation currents on the acoustics in an auditorium is small. The temperature difference between the heated currents and the air in the room is not great enough to affect the sound appreciably, and the motion of the current is too slow and over too short a distance to change the action of the sound to any marked extent.[3]

Under special circumstances, the heating and ventilating systems may prove disadvantageous.[4] A hot stove or a current of hot air in the center of the room will seriously disturb the action of sound. Any irregularity in the air currents so that sheets of cold and heated air fluctuate about the room will also modify the regular action of the sound and produce confusion. The object to be striven for is to keep the air in the room as homogeneous and steady as possible. Hot stoves, radiators, and currents of heated air should be kept near the walls and out of the center of the room. It is of some small advantage to have the ventilation current go in the same direction that the sound is to go, since a wind tends to carry the sound with it.

IV. The Investigation in the Auditorium at the University of Illinois.

A. PRELIMINARY WORK.

As already stated, a chaos of sound was set up when an observer in the Auditorium spoke or shouted or clapped his hands. Both echoes and reverberations were present and could be heard in all parts of the room, though the echoes seemed to be strongest on the stage and in the balcony. The propects for bettering the acoustics were not very encouraging. Luckily, the cure for the reverberation was fairly simple, since Sabine's method gave a definite procedure that could be applied to this case. The cure for the echo, however, was yet to be found. It was first necessary to find out which walls set up the defect.

1. Osborne Reynolds. Proc. of Royal Soc., Vol. XXII, p. 531. 1874.
2. Joseph Henry, "Report of the Lighthouse Board of the United States for the year 1874."
J. Tyndall, Phil. Trans., 1874.
3. Sabine, Engineering Record, Vol. 61, p. 779, 1910.
Watson, Engineering Record, Vol. 67, p. 265. 1913.
4. Sabine and Watson. Ibid.

The attempt to locate echoes by generating a sound and listening with the ear met with only partial success. The ear is sensitive enough, but becomes confused when many echoes are present, coming apparently from every direction, so that the evidence thus obtained is not altogether conclusive. It became apparent that the successful solution lay in fixing the attention on the sound going in a particular direction and finding out where it went after reflection; then tracing out the path in another particular direction, and so on, until the evidence obtained gave some hint of the general action of the sound.

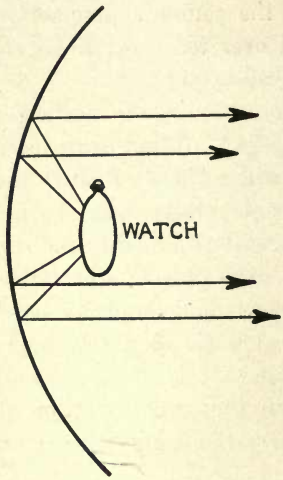

FIG. 4. WATCH AS SOURCE OF SOUND, BACKED BY A CONCAVE REFLECTOR.

The first step in the application of this principle was to use a faint sound which could not be heard at any great distance unless reinforced in some way. The ticks of a watch were directed, by means of a reflector (Fig. 4) to certain walls suspected of giving echoes. Using the relation that the angle of incidence equals the angle of reflection, the reflected sound was readily located, and the watch ticks heard distinctly after they had traveled a total distance as great as 70 to 80 feet from the source.

In a later experiment, a metronome was used which gave a louder sound. It was enclosed in a sound-proof structure (Fig. 5) with only one opening, so that the sound could be directed by means of a horn. This method was suggested by the work of Gustav Lyon in the Hall of the Trocadero at Paris,* where a somewhat similar arrangement was used. The method was successful and verified the observations taken previously.

*La Nature (Paris), April 24, 1909.

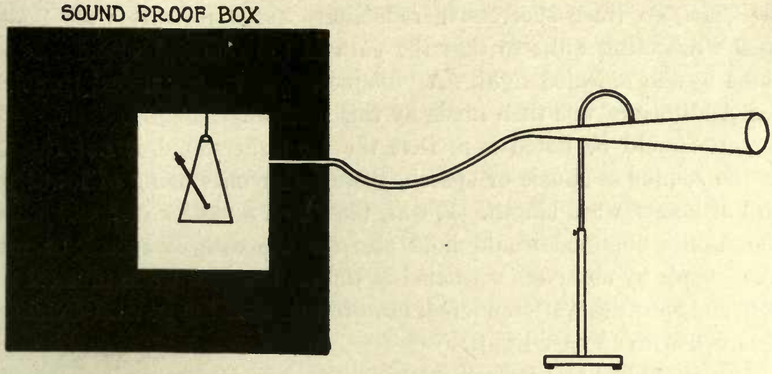

FIG. 5. METRONOME AS SOURCE OF SOUND.

Though the results obtained with the watch and metronome seemed conclusive, yet the observer was not always confident of the results. A further method was sought, and a more satisfactory one found by using an alternating current arc-light at the focus of a parabolic reflector (Fig. 6). In addition to the light, the arc gave forth a hissing sound, which was of short wave length and therefore experienced but little diffraction. The bundle of light rays was, therefore, accompanied by a bundle of sound, both coming from the same source and subject

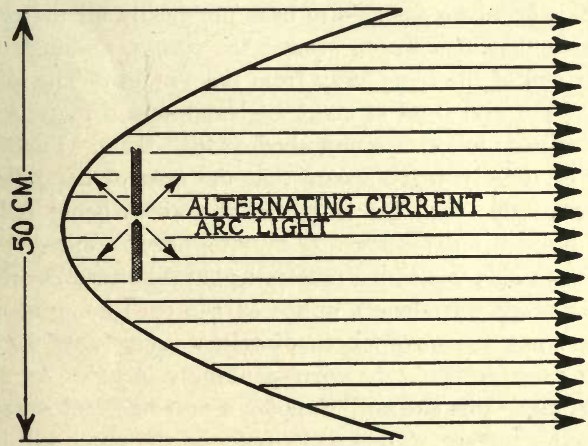

FIG. 6. ARC-LIGHT AS SOURCE OF SOUND.

to the same law of reflection. The path of the sound was easily found by noting the position of the spot of light on the wall. The reflected sound was located by applying the relation that the angles of incidence and reflection are equal. The arc-light sound was intense and gave the observer confidence in results that was lacking in the other

methods. To trace succcessive reflections, small mirrors were fastened to the reflecting walls so that the path of the reflected sound was indicated by the reflected light. A "diagnosis" of the acoustical troubles of the Auditorium was then made by this method.

It should be noted here that the arc-light sound is not the same as the sounds of music or speech, these latter ones being of lower pitch and of longer wave length. It was, therefore, a matter of doubt whether the results obtained would hold also for the case of speech or music. Tests made by observers stationed in the Auditorium when musical numbers and speeches were rendered, however, verified the general conclusions obtained with the arc-light.

It should be pointed out in this connection that there is an objection to applying the "ray" method of geometrical optics to the case of sound. It is much more difficult to get a ray of sound than it is to get a ray of light.* This is due to the difference in the wave lengths in the two cases. It appears that the waves are diffracted, or spread out, in proportion to their length, the longer waves being spread out to a greater extent. The short waves of light from the sun, for instance, as they come through a window mark out a sharp pattern on the floor, which shows that the waves proceed in straight lines with but little diffraction or spreading. Far different is it with the longer waves of sound. If the window is open, we are able to hear practically all the sounds from outdoors, even that of a wagon around the corner, although we may be at the other end of the room away from the window. The longer sound waves spread out and bend at right angles around corners, so that it is almost impossible to get a sound shadow with them. Furthermore, in the matter of reflection, it appears that the area of the reflecting wall must be comparable with the length of the waves being reflected. In the case of light, the waves are very minute, hence a mirror can be very small and yet be able to set up a reflection; but sound waves are of greater length, the average wave length of speech (45 cm.) being about 700 000 times longer than the wave length of yellow light (.00006 cm.), hence the reflecting surface must be correspondingly larger. An illustration will perhaps make this clearer. Suppose a post one foot square projects through a water surface. The small ripples on the water will be reflected easily from the post, but the large water waves pass by almost as if the post were not there. The reflecting surface must have an area comparable with the size of the wave if it is to cause an effective reflection. Relief work in auditoriums, if of small dimensions, will affect only the high pitched sounds, i. e., those of short wave length, while the low

*Rayleigh, "Theory of Sound," Vol. II, § 283.

pitched sounds of long wave length are reflected much the same as from a rather rough wall. It is also shown that the area of the reflecting surface is dependent on its distance from the source of sound and from the observer; the greater these distances are the larger must be the reflecting surface.*

These considerations all show that the reflection of sound is a complicated matter. The dimensions of a wall to reflect sound, or of relief work to scatter it, are determined by the wave length and by the various other factors mentioned. It should be said with caution that a "ray" of sound is reflected in a definite way from a small bit of relief work. We must deal with *bundles* of sound, not too sharply bounded, and have them strike surfaces of considerable area in order to produce reflections with any completeness.

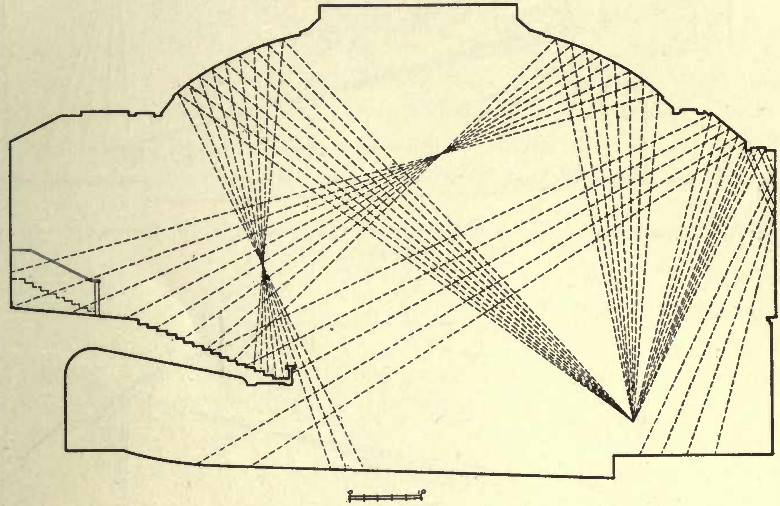

FIG. 7. LONGITUDINAL SECTION SHOWING THE CHIEF CONCENTRATIONS OF SOUND, THE DIFFRACTION EFFECTS BEING DISREGARDED.

B. DETAILS OF THE ACOUSTICAL SURVEY IN THE AUDITORIUM.

The general effect of the walls of the Auditorium on the sound may be anticipated by considering analogous cases in geometrical optics, but with the restrictions on "rays" described in the preceding paragraph. The sound does not actually confine itself to the sharp boundaries shown. The diagrams are intended to indicate the main effect of the sound in the region so bounded. Fig. 7 gives such an idea for the concentration of sound in the longitudinal section of the Auditorium.

*Rayleigh, ibid., 283.

The plan followed in the experimental work was to anticipate the path of the sound as indicated in Fig. 7, then to verify the results with the arc-light reflector. Figs. 8 and 9 show the effect of the rear wall in the balcony in forming echoes on the stage. The speaker was particularly unfortunate, being afflicted with no less than ten echoes.

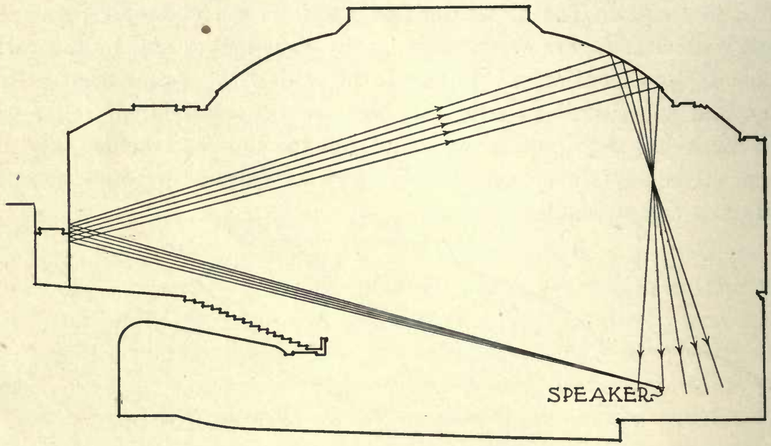

Fig. 8. Longitudinal Section Showing How Sound Is Returned to the Stage to Form an Echo.

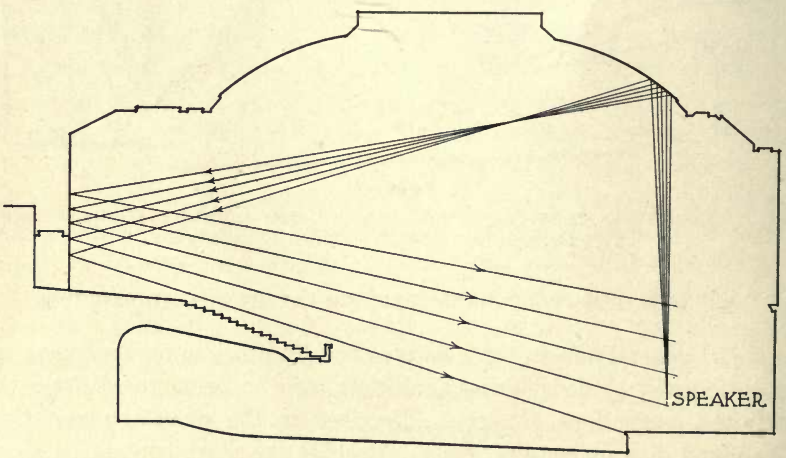

Fig. 9. Longitudinal Section Showing Formation of Echo on the Stage.

The hard, smooth, circular wall bounding the main floor under the balcony gave echoes as shown in Fig. 10, the sound going also in the reverse direction of the arrows.

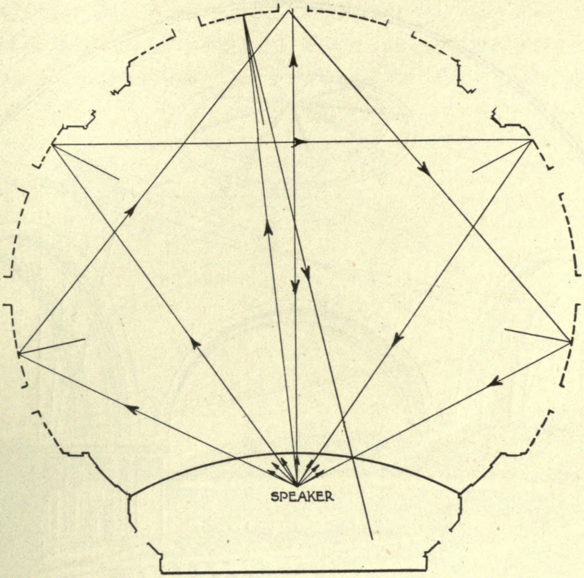

FIG. 10. PLAN OF AUDITORIUM SHOWING ACTION OF REAR WALL ON THE SOUND.

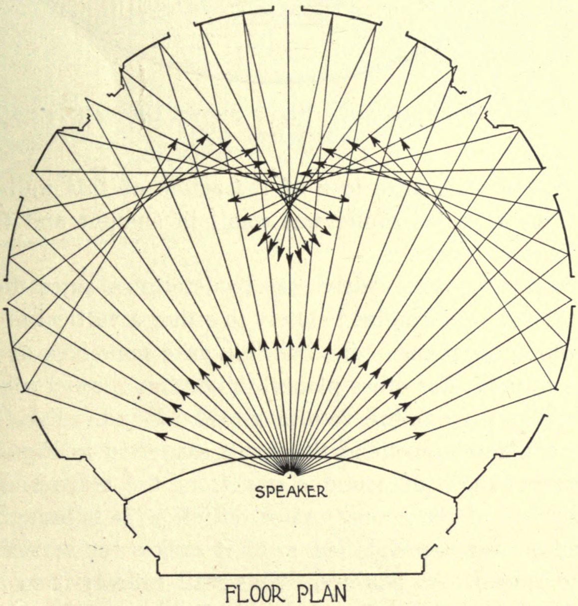

FIG. 11. PLAN OF AUDITORIUM SHOWING CONCENTRATION OF SOUND BY THE REAR WALL.

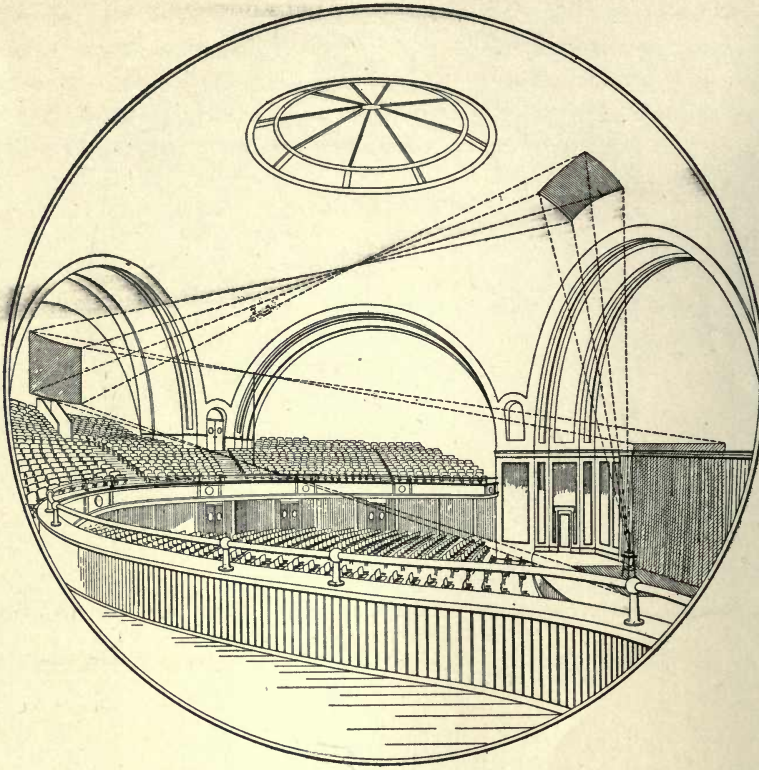

FIG. 12. THIS FIGURE TAKEN WITH FIG. 9 SHOWS HOW AN ECHO IS SET UP ON THE STAGE.

A more comprehensive idea of the action of this wall is shown in Fig. 11. This reflected sound was small in amount and therefore not a serious disadvantage.

The cases cited were fairly easy to determine since the bundles of sound considered were confined closely to either a vertical or a horizontal plane for which the plans of the building gave some idea of the probable path of the sound. For other planes, the paths followed could be anticipated by analogy from the results already found. Fig. 12 shows in perspective the development of the result expressed in Fig. 9.

A square bundle of sound starts from the stage and strikes the spherical surface of the dome. After reflection, it is brought to a point focus, as shown, and spreads out until it strikes the vertical cylindrical wall in the rear of the balcony. This wall reflects it to a line focus, after which it proceeds to the stage. Auditors on all parts of the stage complained of hearing echoes.

Referring to Fig. 7, it is seen that the arch over the stage reflects sound back to the stage. Fig. 13 shows in perspective the focusing action of this overhead arch. Fig. 14 shows the effect of the second arch.

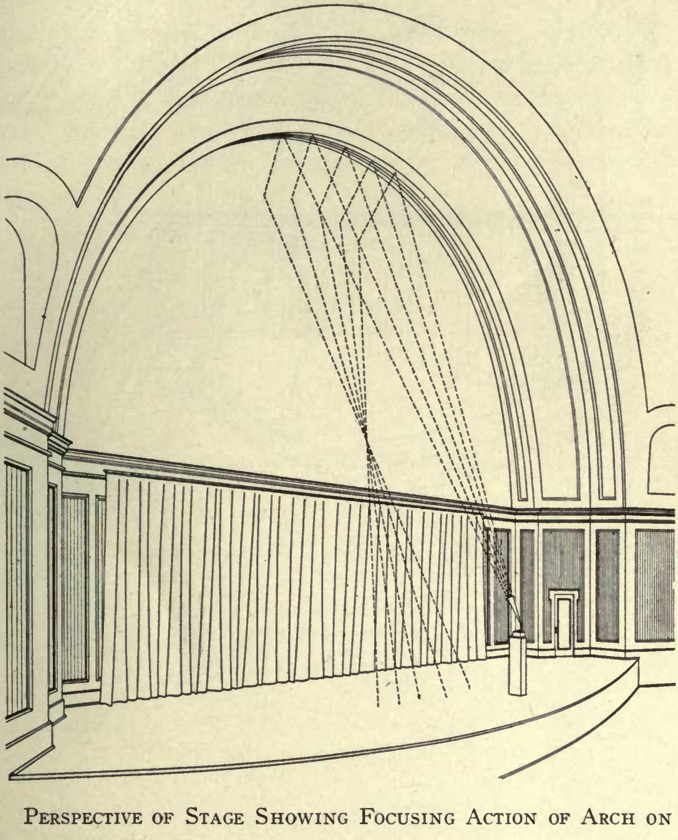

FIG. 13. PERSPECTIVE OF STAGE SHOWING FOCUSING ACTION OF ARCH ON SOUND.

Some of this sound is reflected to the stage and to the seats in front of the stage; other portions, striking more nearly horizontally, are reflected to the side balconies. The echoes are not strong except for high pitched notes with short wave lengths, since the width of the arch is small.

Passing now to the transverse section, Fig. 15, we find the most pronounced echoes in the Auditorium. If an observer generates a sound in the middle of the room directly under the center of the skylight, distinct echoes are set up. A bundle of sound passes to the concave surface which converges the sound to a focus, after which it spreads out again to the other concave surface and is again converged to a focus

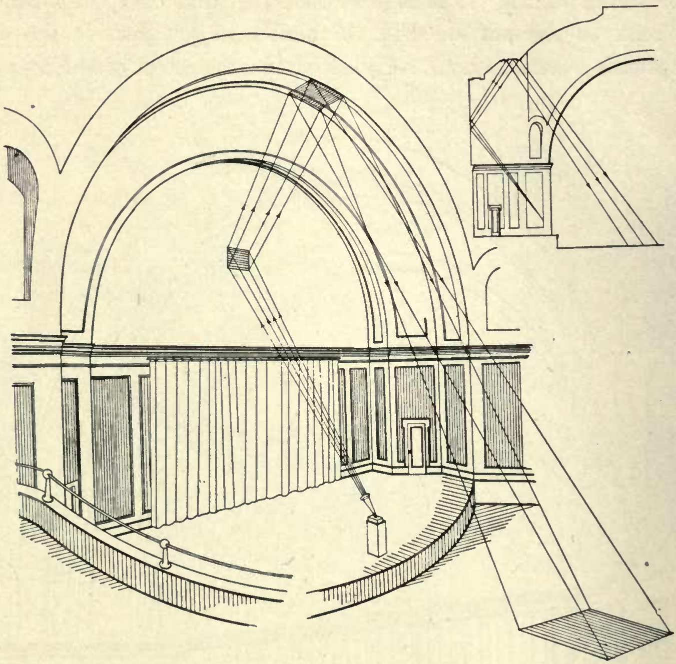

FIG. 14. PERSPECTIVE OF STAGE SHOWING FOCUSING ACTION OF SECOND ARCH.

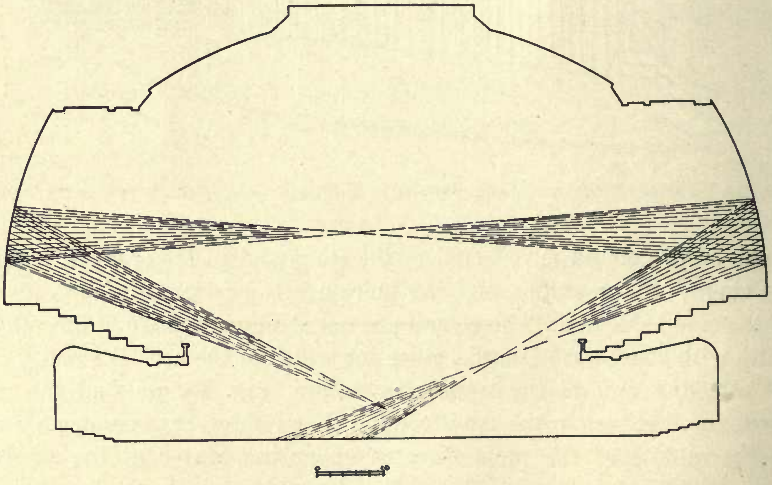

FIG. 15. TRANSVERSE SECTION SHOWING HOW MOST PRONOUNCED ECHOES ARE SET UP BY THE TWO CONCAVE SURFACES.

nearly at the starting point. The distance traveled is about 225 feet, taking about 1/4 second, so that the conditions are right for setting up a strong echo. This echo is duplicated by the sound which goes in the reverse of the path just described. Another echo, somewhat less strong, is formed by the sound that goes to the dome overhead and which is reflected almost straight back, since the observer is nearly at the center of the sphere of which the dome is a part. These echoes repeat themselves, for the sound does not stop on reaching the starting point, but is reflected from the floor and repeats the action just described. As many as ten distinct echoes have been generated by a single impulse of sound.

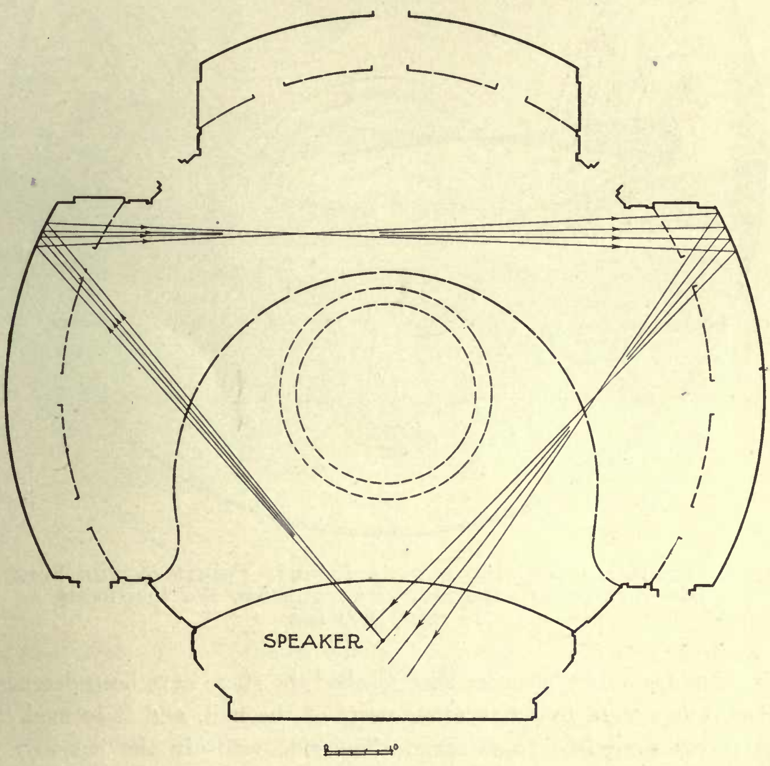

FIG. 16. ACTION OF SOUND IN CAUSING ECHO ON THE STAGE.

The echo shown in Fig. 15 is repeated in a somewhat modified form for a sound generated on the stage by a speaker. Fig. 16 shows the path taken by the sound. This echo is duplicated by the sound that goes in the reverse direction of the arrows, so the speaker is greeted from

both sides. Fig. 17 is a perspective showing the path. The sound does not confine itself closely to a geometrical pattern, as shown in the picture, but spreads out by diffraction. The main effect is shown by the figure.

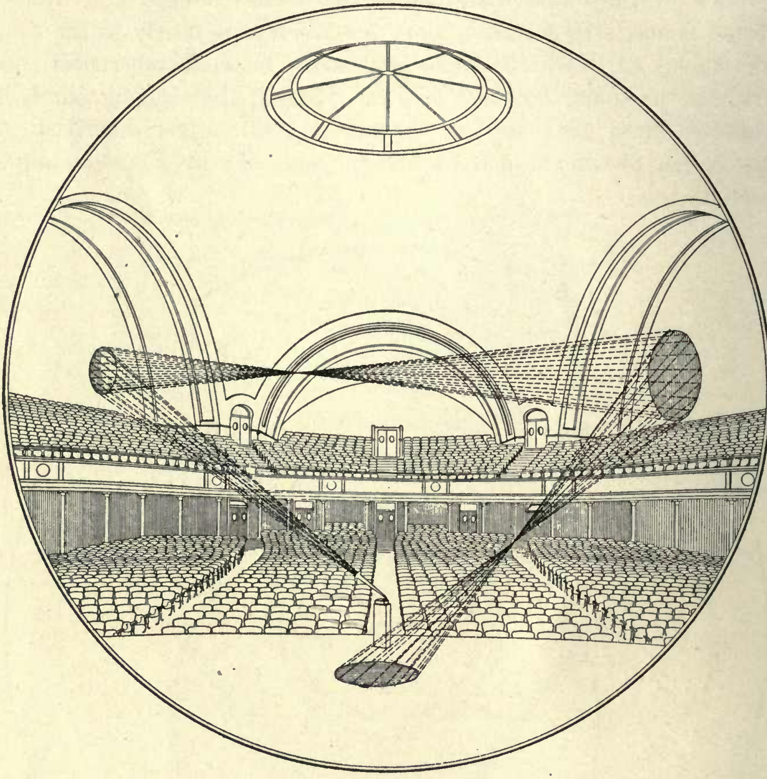

FIG. 17. PERSPECTIVE SHOWING HOW AN ECHO IS FORMED ON THE STAGE BY TWO REFLECTIONS. DIFFRACTION EFFECTS ARE NOT CONSIDERED IN THIS DRAWING.

Thus far only the echoes that reached the stage have been described. Other echoes were found in other parts of the hall, and it seemed that few places were free from them. The side walls in the balcony, for instance, were instrumental in causing strong echoes in the rear of the balcony. Fig. 18 shows in perspective the action of one of these walls. These two surfaces were similar in shape and symmetrically placed. Each was the upper portion of a concave surface with its center of curvature in the center of the building under the dome. The general effect of the left hand wall was to concentrate the sound falling on it

in the right hand seats in the balcony. Some of the sound struck the opposite wall and was reflected to the stage, as shown in Fig. 17. Auditors who sought the furthermost rear seats in the balcony to escape echoes were thus caught by this unexpected action of the sound. The right hand wall acted in a similar way to send the sound to the upper left balcony.

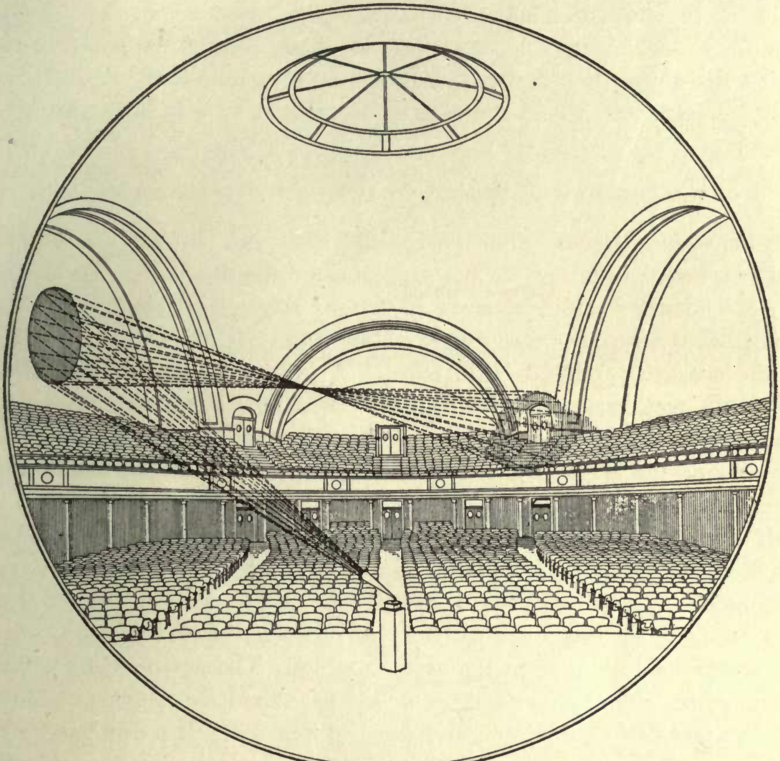

Fig. 18. Perspective Showing Sound Reflector from Concave Wall in Balcony. Diffraction Not Considered.

The dome surface concentrates most of its sound near the front of the central portion of the balcony and the ground floor in front of the balcony in the form of a caustic cone. Figs. 7, 9 and 11 give some conception of how a concentration of sound is caused by this spherical surface. The echo in the front portion of the balcony was especially distinct. On one occasion, in this place, the author was able to hear the speaker more clearly from the echo than by listening to the direct sound.

Minor echoes were set up by the horizontal arch surfaces in the balcony. The sound from the stage was concentrated by reflection from these surfaces and then passed to a second reflection from the concave surfaces back of them. Auditors in the side balcony were thus disagreeably startled by having the sound come from overhead from the rear.

C. CONCLUSION DRAWN FROM THE ACOUSTICAL SURVEY.

The results of the survey show that curved walls are largely responsible for the formation of echoes because they concentrate the reflected sound. It seems desirable, therefore, to emphasize the danger of using such walls unless their action is annulled by absorbing materials or relief work. Large halls with curved walls are almost sure to have acoustical defects.

D. METHODS EMPLOYED TO IMPROVE THE ACOUSTICS.

Reflecting Boards.—The provisional cure was brought about gradually by trying different devices suggested by the diagnosis. In one set of experiments sounding boards of various shapes and sizes were used. A flat board about five feet square placed at an incline over the position of the speaker produced little effect. A larger canvas surface, about 12 by 20 feet, was not much better. A parabolic reflector, however, gave a pronounced effect. This reflector was mounted over a pulpit at one end of the stage and served to intercept much of the sound that otherwise would have gone to the dome and produced echoes. The path of the reflected sound was parallel to the axis of the paraboloid of which the reflector was a quarter section. There was no difficulty in tracing out the reflected sound. Auditors in the path of the reflected rays reported an echo, but auditors in other parts of the Auditorium were remarkably free from the usual troubles. The device was not used permanently, since many speakers objected to the raised platform. Moreover, it was not a complete cure, since it was not suited for band concerts and other events, where the entire stage was used. Another reflector similar in shape to the one just described is shown in Figs. 21 and 22.

Sabine's Method.—The time of reverberation was determined by Sabine's method. An organ pipe making approximately 526 vibrations a second was blown for about three seconds and then stopped. An auditor listened to the decreasing sound, and when it died out made a record electrically on a chronograph drum. The time of reverberation was found to be 5.90 seconds, this being the mean of 19 sets of measurements, each of about 20 observations. The reverberation was found also by calculation from Sabine's equation (see Section III), taking the volume of the Auditorium as 11 800 cubic meters and calculating the

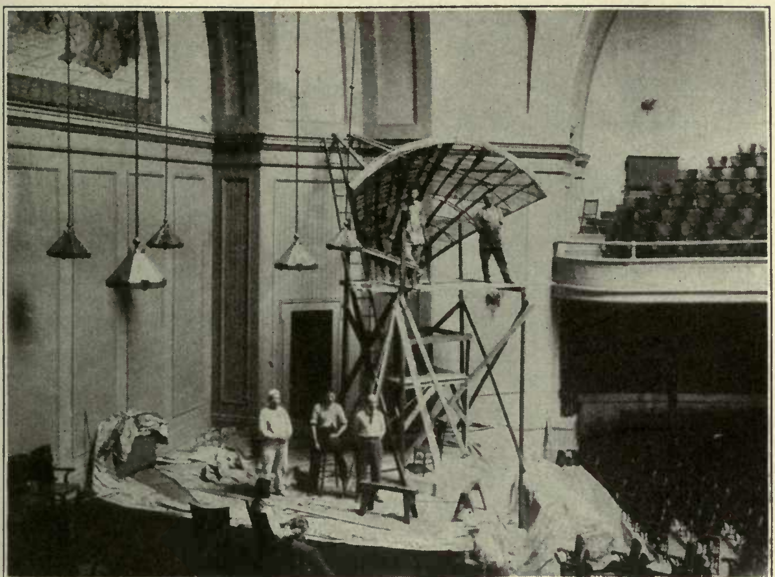

FIG. 19. REFLECTING BOARD IN PROCESS OF CONSTRUCTION.

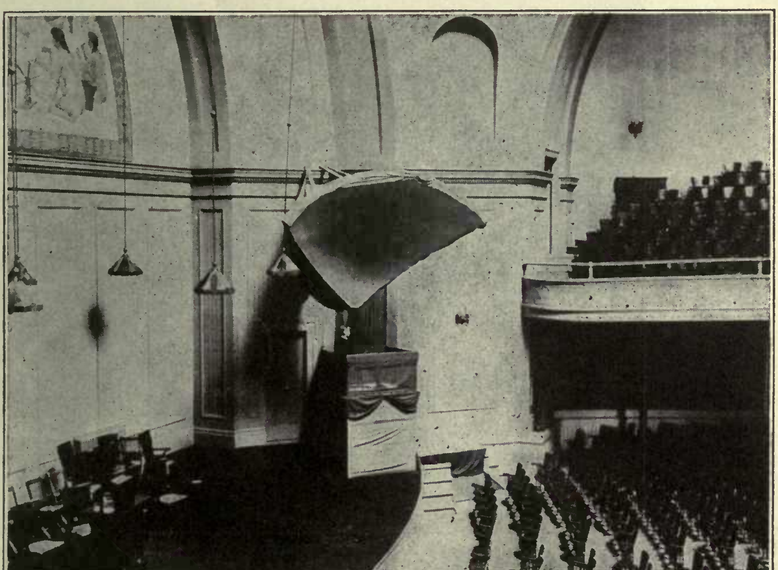

FIG. 20. FINISHED REFLECTOR. HARD PLASTER ON WIRE LATH.

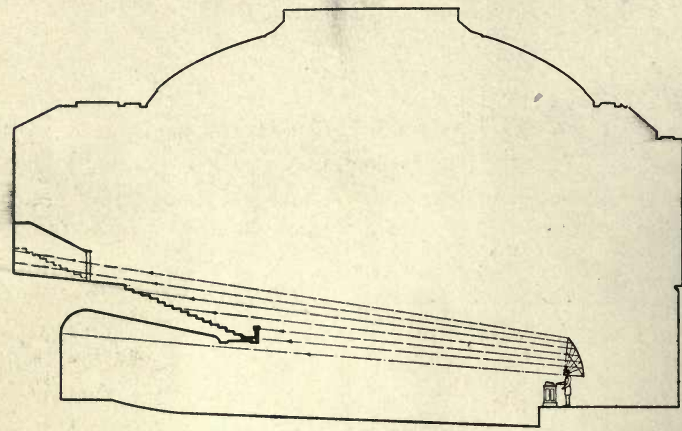

FIG. 21. PARABOLIC REFLECTOR SHOWING ITS ACTION ON SOUND.

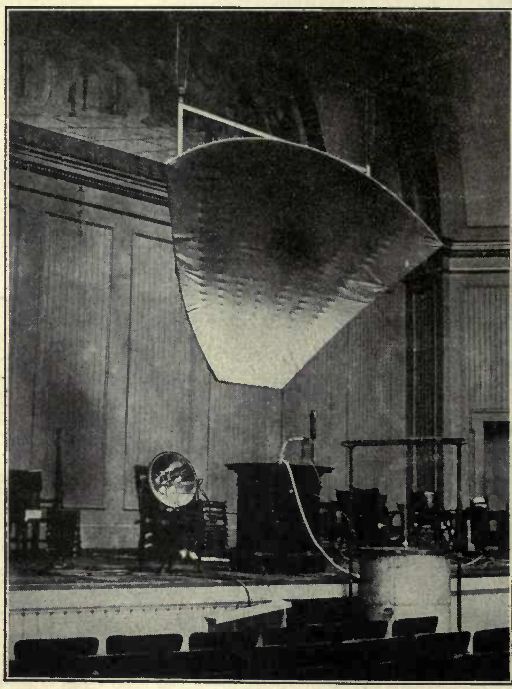

FIG. 22. PHOTOGRAPH OF PARABOLIC REFLECTOR.

absorbing power of all the surfaces in the room. This calculation gave 6.4 seconds. The agreement between the two results is as close as could be expected, since neither the intensity of the sound nor the pitch used by the author was the same as those used by Professor Sabine, and both of these factors affect the time of reverberation.

Several years later the time of reverberation was again determined after certain changes had been made. A thick carpet had been placed on the stage, heavy velour curtains 18 by 32 feet in area hung on the wall at the rear of the stage, a large canvas painting 400 square feet in area was installed, and the glass removed from the skylight in the ceiling. The time of reverberation was reduced to 4.8 seconds. With an audience present this value was reduced still more, and when the hall was crowded at commencement time the reverberation was not troublesome.

Method of Eliminating Echoes.—Although the time of reverberation was reduced to be fairly satisfactory, as just explained, the echoes still persisted, and were very annoying. Attempts were made to reduce individual echoes by hanging cotton flannel on the walls at critical points. Thus the shaded areas in Fig. 17 were covered and also the entire rear wall in the balcony. Pronounced echoes still remained, and it was evident that some drastic action was necessary to alleviate this condition. Four large canvases, shown in Figs. 23 and 24, were then hung in the dome in position suggested by the results of the diagnosis. A very decided improvement followed. For the first time the echoes were reduced to a marked degree and speakers on the stage could talk without the usual annoyance. This arrangement eliminated the echoes not only on the stage, but generally all over the house. A number of minor echoes were still left, but the conditions were much improved, especially when a large audience was present to reduce the reverberation.

Proposed Final Cure.—The state of affairs just described is the condition at the time of writing. Two propositions were considered in planning the final cure. One proposition involved a complete remodeling of the interior of the Auditorium. Plans of an interior were drawn in accordance with the results of the experimental work that would probably give satisfactory acoustics. This proposition was not carried out because of the expense and because it was thought desirable to attempt a cure without changing the shape of the room. The latter plan is the one now being followed. It is proposed to replace the present unsightly curtains with materials which will conform to the architectural features of the Auditorium and which will have a pleasing color scheme. At the same time, it will be necessary to hold to the features which have improved the acoustics.

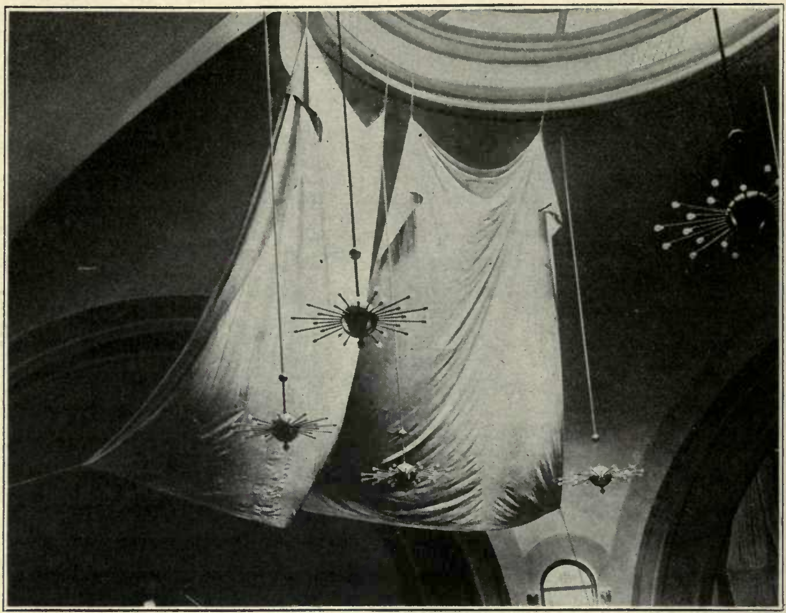

FIG. 23. PHOTOGRAPH OF TWO OF THE CANVAS CURTAINS IN THE DOME OF THE AUDITORIUM. NOTE ALSO THE ABSORBING MATERIALS UNDER THE ARCHES.

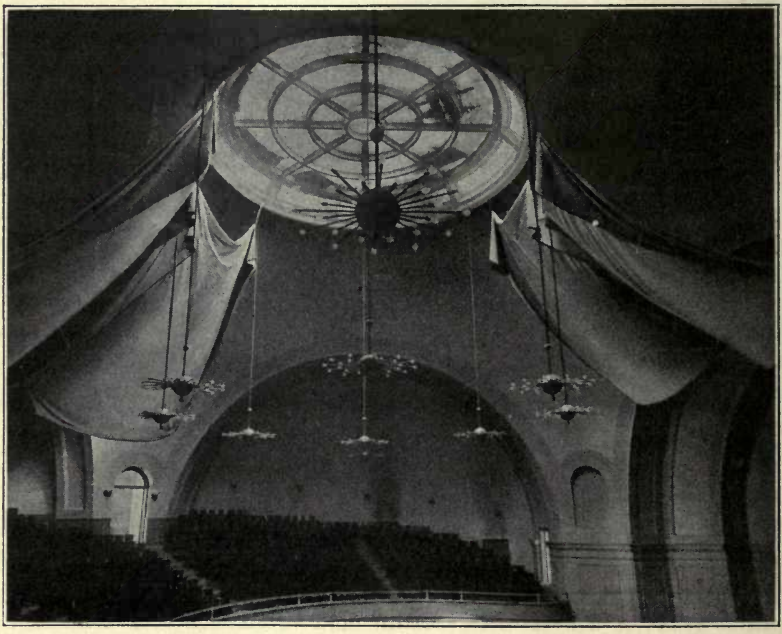

FIG. 24. PHOTOGRAPH OF DOME OF AUDITORIUM SHOWING THE CANVASES INSTALLED TO ELIMINATE ECHOES.

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Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

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A MODIFIED METHOD OF MEASURING *e/m* AND *v* FOR CATHODE RAYS.

BY L. T. JONES.

THIS determination of e/m and v is a modification of the usual method employing the simultaneous electrostatic and magnetic deflections. The modification is the result of an attempt to eliminate as nearly as possible the errors of measurement of the deflections and the correction due to the field distribution at the ends of the electrostatic plates. This is brought about chiefly by the position in which the photographic plate was placed.

THE APPARATUS.

A glass cylinder 10 cm. in diameter and 27 cm. long (Fig. 1) was closed at each end by a glass plate. Two holes were made in one of the plates to admit the glass tubes carrying the anode, A, and the cathode, C. The cathode, an aluminum disc .6 cm. in diameter, was carried on an aluminum rod. This rod was encased in a small glass tube which in turn was supported by a larger glass tube waxed to the glass plate where it entered the discharge chamber. The anode was mounted in a similar manner. Both aluminum rods were connected with the outside by platinum wires sealed in glass.

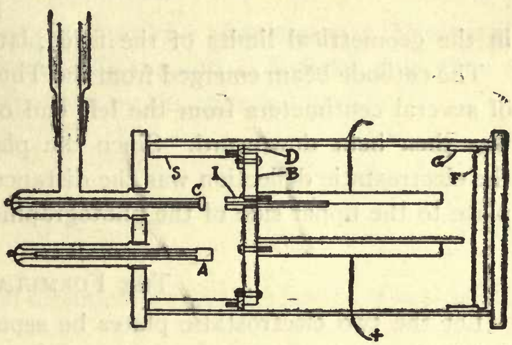


Fig. 1.

A brass ring, D, was fastened by sealing wax to the inside of the glass cylinder, and to this were fastened the soft iron shield, S, and the ebonite disc, B, the latter supporting the electrostatic plates. The electrostatic plates were held to the disc B by brass screws. The potential of the electrostatic plates was supplied through two wires that passed through small holes in the walls of the cylinder. The holes were sealed with wax.

By loosening the screws holding the ebonite disc, B, to the brass ring, D, the disc and electrostatic plates could be taken as a whole from the cylinder.

At the opposite end a short length of brass cylinder, G, was waxed to the inside of the glass cylinder and a hard rubber disc, F, turned to fit it, darkened the tube. The glass cylinder was coated on the outside with lamp black and the coating connected to earth. All the metal parts inside the tube, except the electrostatic plates and the discharge terminals, were connected to earth.

The Electrostatic Plates.

Two electrostatic plates were mounted exactly 1 cm. apart, as shown in Fig. 2. The beam of cathode rays was made to pass along the upper of the two plates at grazing incidence. The photographic plate was placed flat on the lower of the two electrostatic plates. The beam was bent downward, by adjusting the electric field, to strike the photographic plate well within the geometrical limits of the field plates.

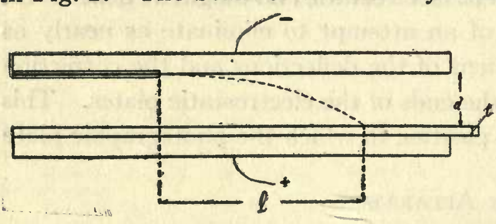


Fig. 2.

The cathode beam emerged from the Thomson plate-tube at a distance of several centimeters from the left end of the electrostatic plates and was then bent downward. Since the plates were plane and parallel the electrostatic deflection was the distance from the upper electrostatic plate to the upper side of the photographic plate.

The Formula.

Let the two electrostatic plates be separated by a distance $d + t$, d being the air place and t the thickness of the photographic plate, which is of dielectric constant K. The two electrostatic plates are kept at constant potentials V' and V''.

If the plates are separated by an air space of thickness $d + t$ there is a given electric surface density of charge on the plates and, consequently, a given electric force, E, in the space between them. If, now, the dielectric of thickness t is introduced, whose equivalent air thickness is t/K, the effective air space will then be reduced from $d + t$ to $d + t/K$. The effective air space has thereby been reduced an air-equivalent amount of $t - t/K$, causing a change in the capacity. Since the potentials of the two plates have remained constant the surface density and hence the

electric force have changed. If, now, the air gap between the two electrostatic plates is increased by an amount $t - t/K$, then the capacity, the surface density and the electric force will resume their former values. If, then, while an electric force, E, exists between the two plates of potentials V' and V'', a dielectric slab of thickness t is introduced and at the same time the plates are further separated by an amount $t - t/K$, making $d + 2t - t/K$ in all, the surface density on the plates and hence the electric force, E, will remain constant. The electric force is then given by the equation

$$V' - V'' = E(d + 2t - t/k),$$

or

$$E = \frac{PD \times 10^8}{d + 2t - t/k}, \tag{1}$$

where PD is the potential difference of the two electrostatic plates in volts.

The cathode beam in passing through the uniform electric field, E, is accelerated by a constant force and hence follows Newton's second law. The force on the charge e will be

$$Ee = ma, \tag{2}$$

where a is the acceleration toward the positive plate and m is the mass of the electron. Since the electron falls through a distance d in time t we have the distance of fall expressed by the equation

$$d = {}^1/_2 at^2,$$

or

$$a = \frac{2d}{t^2}. \tag{3}$$

If the velocity in the horizontal direction is v and the length of horizontal travel is l we have

$$l = vt,$$

whence

$$\frac{1}{t^2} = \frac{v^2}{l^2}.$$

Substituting this value in equation (3) gives

$$a = \frac{2dv^2}{l^2}.$$

If this value is placed in equation (2) we have

$$Ee = \frac{2dmv^2}{l^2},$$

whence

$$\frac{e}{mv^2} = \frac{2d}{El^2}. \tag{4}$$

If at the same time the moving electron is subjected to the action of a uniform magnetic field of intensity H and its velocity v is perpendicular to the lines of magnetic force, urging the particle in the path of a circle, in the plane of the photographic plate, then the force is given by

$$Hev = \frac{mv^2}{r} \tag{5}$$

where r is the radius of the circle. If the dotted line, Fig. 3, indicates the path of the particle undeflected by the magnetic field, and the circle of radius r the curvature experienced under the influence of the magnetic field of strength H we may represent the horizontal distance traveled by the length l and the magnetic deflection (measured at right angles to the undeflected path) by z, since z is small compared with l. Then, from Fig. 3,

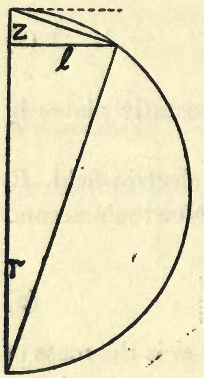


Fig. 3.

$$\frac{z}{\sqrt{z^2 + l^2}} = \frac{\sqrt{z^2 + l^2}}{2r}$$

and

$$\frac{1}{r} = \frac{2z}{l^2},$$

since z^2, being small in comparison with l^2, may be neglected. Placing this value of $1/r$ in equation (5) we have

$$\frac{e}{mv} = \frac{1}{Hr} = \frac{2z}{Hl^2}. \tag{6}$$

Elimination of e/m between (4) and (6) gives

$$v = \frac{zE}{Hd}.$$

Replacing E by its value given in (1) we get, after simplification,

$$v = \frac{zPD \times 10^8}{Hd(d + 2t - t/K)}. \tag{7}$$

Again, multiplying equations (6) and (7) gives

$$\frac{e}{m} = \frac{z^2PD \times 2 \times 10^8}{H^2l^2d(d + 2t - t/K)}. \tag{8}$$

The Electrostatic Field.

The two electrostatic plates were rectangular brass plates $7.5 \times 15 \times 1$ cm. Considerable difficulty was experienced in getting the two plates

sufficiently plane. The plates were first planed and then finished by "spotting" on a master plate. A slip of soft iron $5 \times 1.5 \times .15$ cm. was inlaid in the upper plate, as shown in Fig. 2, and the plate again surfaced. Several days were required to surface the plates but they were finally finished sufficiently plane that one would raise the other from the table.

A second slip of soft iron was cut out $5 \times 1.5 \times .1$ cm. and one side made plane. A scratch .005 cm. in width and of about the same depth was drawn full length on this surfaced side. This scratch formed the tube through which the cathode rays passed. The iron slip with the scratch was held against the iron slip inlaid in the upper electrostatic plate by ten brass screws. On account of the small diameter of the scratch and its relatively large length it was subsequently found to be easier to make a scratch of about .05 cm. in diameter, close each end with a small bit of solder, cut off the solder flush with the iron surface and then make a small scratch in the bit of solder at each end. A scratch .1 cm. long at each end was found to give perfect satisfaction, and not nearly so much difficulty was experienced in getting the beam to pass through this tube. In adjusting the cathode to send a beam through the tube the electrostatic plates were first mounted in position with the scratch the full .05 cm. diameter. The vessel was exhausted and a potential difference of about 20 volts applied to the electrostatic plates. The wax joint where the glass tube supporting the cathode entered the plate glass end was then softened by heating and the cathode moved about until a phosphorescent spot on the willemite screen, deposited on the opposite glass end plate, showed the presence of the beam. The wax was allowed to cool while the cathode was in the position giving this spot its maximum brightness. The electrostatic plates were then removed by taking out the screws holding the ebonite disc, B, to the brass ring, D, and the tube made smaller by the bits of solder mentioned above. The plates were then replaced in position and the vessel exhausted. If the spot failed to show on the willemite screen the process was repeated until finally the beam was made to pass through the small tube.

An iron tube, I, .5 cm. diameter and 2 cm. long, was screwed into the disc, B, to shield the rays from any magnetic effect before entering the confining tube. The cathode was within 1 cm. of the tube I.

The electrostatic plates were spaced by four hollow ebonite cylinders, one placed at each corner, and clamped in position by ebonite bolts passing through the cylinders. The length of these cylinders was measured by a micrometer caliper reading to .001 cm. The cylinder was placed between two thin glass plates and the length of the whole

measured. The thickness of the plates was then subtracted. Each cylinder was measured on several successive days and the mean of these measurements was taken as the length. When the cylinders were again measured, after having been in the apparatus under pressure for four months, they were found to have shortened by about 1 per cent. All data was taken during the first fifteen days, however, so no correction was made for this change in length. The potential difference of the electrostatic plates was determined as follows: A high potential storage battery, *T*, was used in sending a small current through the two high resistances, *M* and *R*, as shown in Fig. 4. *M* was a resistance of about 2×10^6 ohms while *R* was an adjustable resistance of about 10,000 ohms. The electrostatic plates were connected directly to the terminals of *M* as shown. By adjusting the value of *R* the potential difference of the terminals of *M* could be kept constant. The potential drop through a small part of *M* was measured by a potentiometer, *P*, against a Weston standard cell of 1.0185 volts at 24° C. The potential difference of the electrostatic plates was thus easily measured to .1 per cent. and by means of *R* the value was kept constant to within .1 volt.

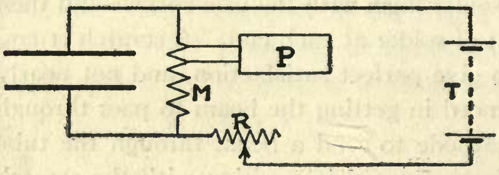


Fig. 4.

THE MAGNETIC FIELD.

The magnetic field was furnished by a solenoid of 648 turns and 160.2 cm. length. The solenoid was built in two parts and made to join closely at the middle so as to enclose the whole tube. The length of the solenoid was such that the field could be considered uniform and calculated. From the dimensions of the solenoid the strength of the magnetic field at its center was given by

$$H = 5.083\, I,$$

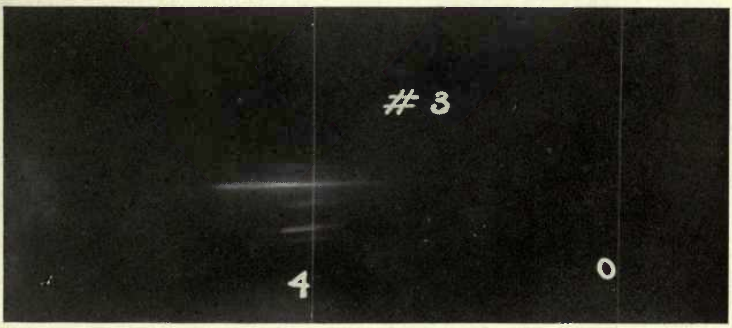
where *I* is the strength of the current in amperes. The current for the magnetic field was supplied by storage cells of 40 amperes capacity. The current, which varied between .5 and 1.5 amperes, was measured by a Siemens & Halske ammeter reading to .005 amperes.

RESULTS.

In placing the photographic plate in the apparatus for exposure the plate was placed solidly against the ebonite disc, *B*. The iron confining tube for the cathode beam was 5.08 cm. long and hence a line drawn

No. 18.

No. 6.



No. 3.

Fig. 5.

L. T. JONES.

across the plate 5.08 cm. from the end that touched the ebonite disc established the zero. This line, marked *O* in the photographs, was then directly under the opening of the tube. The length of horizontal travel, *l*, was measured from this line. In photograph 6 two calculations of *e/m* were made, where the distance *l* was 4 and 5 cm. respectively. In each photograph the long streamer, second from the top, is the central one, given by zero magnetic field. The two spots immediately on either side are for the magnetic deflection, direct and reversed. The additional spots seen have no significance relative to the value of *e/m*. The magnetic deflections were accurately measured along the lines drawn parallel to the line marked *O*. The reproductions in Fig. 5 are full size. Twenty photographs were taken in succession. Table I. gives the data relative to all these.

Table I.

Plate No.	I.	PD.	z.	l.	d.	t.	$d+2t-\frac{t}{K}$.	$v \times 10^{-9}$.	$\frac{e}{m} \times 10^{-7}$.
1	.460	524.0	.138	4.0	.835	.165	1.292	2.866	2.114
2	.450	564.4	.127	4.0	.820	.180	1.210	3.158	2.192
3	.890	498.1	.245	4.0	.825	.175	1.204	2.715	1.838
4	.8902	639.7	.220	4.0	.830	.160	1.177	3.183	1.935
6	.885	425.0	.251	4.0	.811	.179	1.199	2.438	1.701
6a	.885	425.0	.3116	5.0	.811	.179	1.199	3.027	1.678
7a	.850	323.5	.325	5.0	.805	.185	1.206	2.506	1.508
7b	.850	323.5	.371	5.5	.805	.185	1.206	2.861	1.624
7c	.850	323.5	.397	6.0	.805	.185	1.206	3.061	1.563
8a	.8725	323.0	.314	4.5	.805	.185	1.206	2.355	1.647
8b	.8725	323.0	.372	5.5	.805	.185	1.206	2.790	1.547
9	.879	320.3	.335	5.0	.810	.180	1.200	2.470	1.482
10a	.864	301.0	.389	5.0	.825	.165	1.182	2.734	1.937
10b	.864	301.0	.423	6.0	.825	.165	1.182	2.973	1.591
11	.867	299.0	.377	5.0	.826	.164	1.181	2.622	1.807
12a	.873	298.0	.405	5.0	.826	.164	1.181	2.788	2.036
12b	.873	298.0	.461	6.0	.826	.164	1.181	3.173	1.832
13	.872	298.0	.382	5.0	.824	.166	1.184	2.632	1.815
14a	.874	284.2	.4335	5.5	.819	.171	1.189	2.947	1.837
14b	.874	284.2	.499	6.5	.819	.171	1.189	3.278	1.735
15	.881	247.2	.467	6.0	.817	.173	1.192	2.647	1.533
17	.450	248.6							
18	.453	246.6							
19	.4515	245.3							
20	1.289	296.7	.558	5.3	.850	.140	1.153	2.578	1.563
Average									1.748

The value of the dielectric constant of the glass plate was that given by Landolt and Börnstein for "spiegel glas." If the value of *K* was taken as either 5 or 7 instead of 6 the resulting value of *e/m* is changed by only

about .5 per cent. The probable error of the final result, calculated in the usual way from the data in Table I., is 1.5 per cent.

Summary

The method devised for the determination of e/m and v for cathode rays from a cold cathode is a modification of the usual electrostatic and magnetic deflection photographic method. It has two distinct advantages.

1. Both the electrostatic and magnetic fields are uniform over the entire path of the deflected cathode beam.

2. The electrostatic deflection is kept constant for all strengths of fields employed and thus the inaccuracy in its measurement is eliminated.

The mean of twenty successive photographs gave

$$e/m = 1.75 \pm .03 \times 10^7.$$

I wish to express my appreciation to Dr. C. T. Knipp for his kindly suggestions and to Professor A. P. Carman, Director of the Laboratory, for the facilities offered.

Laboratory of Physics,
University of Illinois,
January 20, 1914.

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Pages 555-562.

THE DETERMINATION OF E/M FOR CATHODE RAYS AS A LABORATORY EXPERIMENT FOR AN UNDERGRADUATE COURSE IN ELECTRICAL MEASUREMENTS.

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The numerical value of the ratio of the charge to the mass of the cathode ray particle or electron is no longer a problem for the research laboratory. It is as truly a constant as is the heat of fusion of ice, or Joule's mechanical equivalent, and its value is nearly as accurately known. However, its experimental determination is generally considered to be fraught with manipulative difficulties of considerable moment, which under the ordinary conditions of laboratory equipment makes it an extremely difficult, if not an impossible, experiment for undergraduates.

The recent development of the electron theory with its bearing upon the nature of electricity and the probable constitution of matter is so fundamental and far-reaching that it seems quite proper that one or more of these comparatively new and yet apparently more difficult experiments should be included in an undergraduate course in electrical measurements. Hence the following elementary theory for the determination of e/m for cathode rays is presented as an instructive and practical experiment. For pedagogical reasons the discussion is given as recently presented to a class in electrical measurements for juniors. The apparatus necessary consists of a Braun tube having a pair of electrostatic field plates inside, a three-inch induction coil, and a high potential battery capable of furnishing potential differences up to 500 volts, also a commutator, a simple water resistance and a voltmeter.

The more important characteristic properties of the cathode rays are—they travel in straight lines and have a high velocity, the electrons composing the rays each carry a negative elementary charge, they are deflected by either a magnetic or an electrostatic field, they apparently have a mass that is 1/1700 that of the hydrogen atom, they may ionize a gas through which they pass.

and upon striking an object they cause that object to emit Roentgen rays.

The charge on the electron and the mass of the electron are severally quite difficult to determine. However, their ratio, e/m, is comparatively easy of measurement. The velocity of the electron also admits of easy measurement. In the theory that follows, the property of the magnetic and electrostatic deflection of these rays is employed.

(a.) ELECTROSTATIC DEFLECTION OF A MOVING ELECTRON.[1]

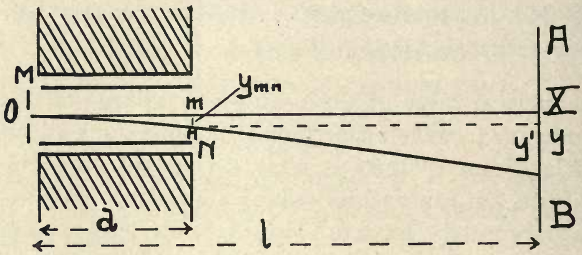

FIGURE 1.

In Figure 1 let

OX = path of undeflected beam.
MN = electrostatic field plates.
AB = screen.
Y = electric force per cm. between plates.
d = length of field plates.
l = distance of screen from opening O in diaphragm.
y = electrostatic deflection on screen.

Let the lower plate be charged positively and the upper one negatively, and suppose that the field ends abruptly at the edge of the plates (the error is quite negligible if the plates are close together). The electron in moving through the uniform electric field is urged toward the positive plate with a force Ye, and its equation of motion is

$$m\frac{d^2y}{dt^2} = Ye$$

where $\frac{d^2y}{dt^2}$ is its acceleration in the y-direction. From which

$$\text{Accel.} = \frac{Ye}{m} = a.$$

Applying the laws of falling bodies, the deflection becomes

$$y_{mn} = \frac{1}{2}at^2 = \frac{1}{2} \cdot \frac{Ye}{m} \cdot \frac{d^2}{v^2},$$

[1]Thomson, *Conduction of Electricity through Gases*, 2d Ed., p. 117.

and the velocity downward is given by

$$v_{mn} = at = \frac{Ye}{m} \cdot \frac{d}{v} \, .$$

The path of the electron from the edge of the field plates to the screen AB is a straight line, hence the additional deflection downward in traveling this distance is

$$y' = v_{mn} \times t' = \frac{Ye}{m} \cdot \frac{d}{v} \cdot \frac{l-d}{v} \, .$$

Therefore the whole distance downward is

$$y = y_{mn} + y' = \frac{1}{2} \cdot \frac{Ye}{m} \cdot \frac{d^2}{v^2} + \frac{Ye}{m} \cdot \frac{d}{v} \cdot \frac{l-d}{v}$$

$$= \frac{Ye}{mv^2} \cdot d\left(l - \frac{d}{2}\right),$$

which may be written,

$$y = AY\frac{e}{mv^2}, \qquad \textbf{(1)}$$

where A is a constant depending upon the geometrical data of the discharge tube. It should be noted that the electrostatic deflection is inversely proportional to the energy of the moving electron.

(b.) MAGNETIC DEFLECTION OF A MOVING ELECTRON.

For convenience in discussing make the magnetic field coterminous with the electric field. Let the shaded portions in Figure 1 represent the pole pieces. The magnetic lines are then parallel to electric lines of force. As in the electric case, consider that the magnetic field ends abruptly at the edge of the pole faces.

It was shown by Rowland and others that a moving charged particle is equivalent to a current of electricity, or

$$i = ev,$$

where e is the charge on the particle and v its velocity. A conductor carrying a current in a magnetic field is urged by a force at right angles to both the direction of field and current. The magnitude of this force is

$$F = Hi = Hev.$$

In a uniform magnetic field the electron, under the action of a constant force at right angles to its direction of motion, moves in a circular path. By a theorem in mechanics its normal acceleration towards the center is

$$a' = \frac{v^2}{\rho},$$

where ρ is the radius of the circle; and the force towards the center is equal to

$$\text{mass} \times \text{accel.} = \frac{mv^2}{\rho}.$$

Hence the equation of motion for a moving electron in a uniform magnetic field is

$$\frac{mv^2}{\rho} = \text{H}ev,$$

or

$$\frac{1}{\rho} = \frac{\text{H}e}{mv}.$$

This force urges the electron in the z-direction, *i. e.,* in a direction perpendicular to the plane of the paper. To evaluate $1/\rho$ construct a circle of diameter 2ρ and lay off d, the length of pole face, and z_{mn}, the magnetic deflection, as shown in Figure 2. Draw the additional lines indicated. Then, from the similar right angled triangles, it follows that

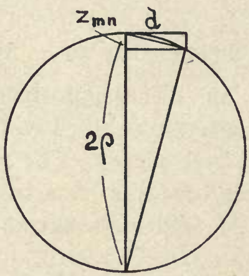

FIGURE 2.

$$\frac{z_{mn}}{\sqrt{z^2_{mn}+d^2}} = \frac{\sqrt{z^2_{mn}+d^2}}{2\rho},$$

from which

$$z_{mn} = \frac{z^2_{mn}+d^2}{2\rho} = \frac{d^2}{2\rho} \text{ approximately,}$$

since z_{mn} is small in comparison with d. Therefore

$$z_{mn} = \frac{1}{2} \cdot \frac{\text{H}e}{mv} \cdot d^2.$$

Following the same line of development as in the electrostatic case, the velocity at m, in the z-direction is given by

$$v'_{mn} = a' t = \frac{v^2}{\rho} \cdot \frac{d}{v} = \frac{Hed}{m} .$$

The additional magnetic deflection is

$$z' = v'_{mn} t' = \frac{Hed}{m} \cdot \frac{l-d}{v} .$$

Hence the total magnetic deflection becomes

$$z = z'_{mn} + z' = \frac{1}{2} \cdot \frac{He}{mv} \cdot d^2 + \frac{Hed}{m} \cdot \frac{l-d}{v}$$

$$= \frac{He}{mv} \cdot d \left(l - \frac{d}{2}\right) ,$$

which may be written

$$z = B \cdot \frac{He}{mv} . \qquad (2)$$

where B, as in the electric case, is a constant depending upon the geometrical data of the discharge tube. The magnetic deflection is inversely proportional to the momentum of the moving electron. It should be remarked that both equations (1) and (2) are true for heavy carriers (atomic or molecular) having either positive or negative charges.

For coterminous fields

$$A = B.$$

By applying the two fields simultaneously and at the same time giving them the proper directions the spot on the screen will take up its position at some point P, whose co-ordinates are y, z. Obviously, under these conditions, either the velocity v, or the ratio e/m may be calculated by combining the two equations. Eliminating v between (1) and (2) gives

$$v = \frac{AYe}{mv} \cdot \frac{1}{y} = \frac{BHe}{m} \cdot \frac{1}{z} ,$$

from which

$$v = \frac{AY}{BH} \cdot \frac{z}{y} . \qquad (3)$$

Substituting this value of v in (2) and solving gives for the ratio of the charge to the mass,

$$\frac{e}{m} = \frac{AY}{B^2H^2} \cdot \frac{z^2}{y} . \qquad (4)$$

Application to the Braun Tube.

This experiment may be performed in the case of the Braun tube, by using the intensity of the *earth's* magnetism in place of

the usual solenoid or electromagnet. The relative position, for the particular tube used, of the aperature O, the electric field plates MN, and the screen AB is shown in Figure 3. From the dimensions given

$$A = d\left(l - \frac{d}{2}\right) = 8\left(32 - \frac{8}{2}\right) = 2.24 \times 10^2,$$

and

$$Y = \frac{V \times 10^8}{2.4},$$

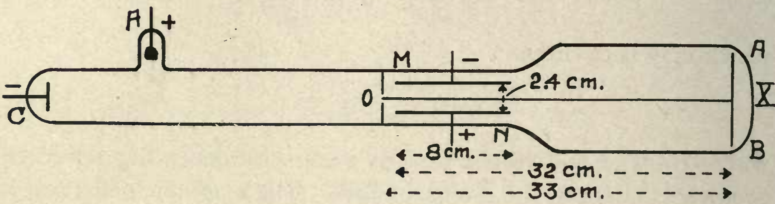

FIGURE 3.

where V is the potential difference in volts between the plates M and N.

Again, the constant B becomes, since the magnetic field acts over the entire distance OX and hence $d = l$,

$$B = \frac{l^2}{2} = \frac{33^2}{2} = 5.445 \times 10^2,$$

and

$$B^2 = 2.96 \times 10^5.$$

Equations (3) and (4) thus become, for this particular tube,

$$v = 1.71 \frac{V}{H} \cdot \frac{z}{y} \cdot 10^7 \tag{3'}$$

and

$$\frac{e}{m} = 3.15 \frac{V}{H^2} \cdot \frac{z^2}{y} \cdot 10^4 \tag{4'}$$

where H, as suggested above, is the magnetic field due to the earth.

Now for H either the total intensity or the horizontal component may be used. The former gives the larger deflection, however the difficulty of inclining the tube at the proper angle is considerable, especially within a building having an iron framework, hence it is more reliable and the experiment easier to perform when using the horizontal component. The vertical component, while it displaces the spot on the screen to the north or

south (depending on the orientation of the tube), introduces no error since the direction of the electrostatic field is reversed in taking readings for each position of the tube. To enable the tube to be placed in these various positions quickly and accurately it should be mounted on a wooden frame that will admit of rotation of the tube about a vertical and also a horizontal axis, as shown in Figure 4. The value of H (the horizontal component) at the point in the laboratory where the experiment was performed was previously determined by comparison with the value of H out in the open and about one mile from local magnetic disturbances, and was found to be

$$H = .160$$

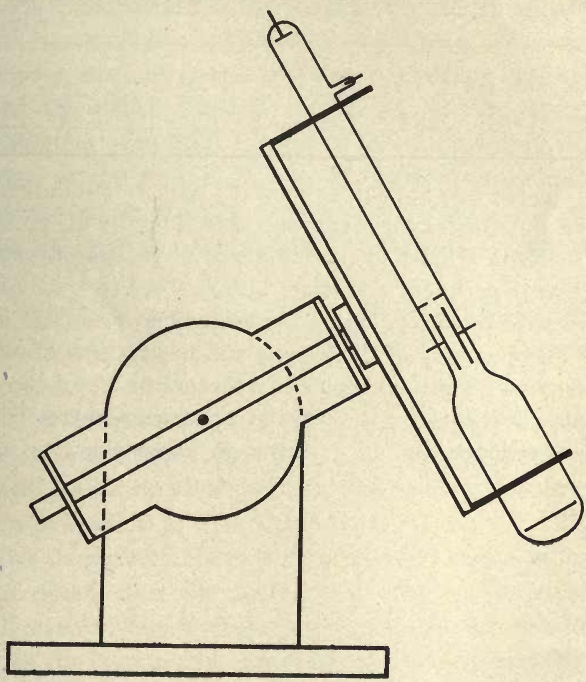

FIGURE 4.

Typical sets of data are contained in Tables I and II. The conditions that obtained in the two sets were the same with one important exception. In Table II the precaution was taken to shield that portion of the Braun tube between the cathode and the diaphragm O from the earth's magnetic field by wrapping it with two layers of annealed sheet iron. This shield was connected to earth. The effect of the shielding was quite noticeable in that the spot on the screen was brighter and steadier, thus enabling the deflections

to be more accurately read. The values of v are considerable in excess of those obtained by more refined apparatus, while the values of e/m agree indeed very closely with the generally accepted value which is 1.77×10^7. In fact the apparatus scarcely warrants such close agreement, yet repeated observations with widely different voltages gave values for e/m in excess but a few per cent of the true value.

TABLE I.

Readings on Screen AB. Bulb to West Direct		Bulb to West Reversed		Bulb to East Direct		Bulb to East Reversed		Mean Mag. defl. in cm.	Mean Electric defl. in cm.	V in Volts	$v \times 10^{-9}$	$e/m \times 10^{-7}$
Z	Y	Z	Y	Z	Y	Z	Y	z	y			
48	24	48	33	51	24	51	34	.15	.475	190	6.4	1.10
47	17	47	38	51	17	51	38	.2	1.05	372	7.5	1.74
47	14	47	43	51[2]	8[2]	51	43	.2	1.60	542	7.2	1.66
											Average,	1.50
TABLE II.												
42	27	42	37.5	46	27.5	46	38	.2	.525	206	8.4	1.93
42.5	20	42.5	42	46.5	19	47.5	42	.225	1.125	409	8.7	1.78
42	14	43	47.5	47	13	48[2]	47.5	.25	1.70	625	9.8	1.84
											Average,	1.85

In conclusion it is but fair to mention that the preliminary work of testing out this experiment was ably done by F. E. Faulkner and E. A. Reid, seniors in the University. It was presented later as a regular experiment (on trial) before three sections of juniors in electrical engineeering as an experiment in electrical measurements. The accuracy of the results and the favor with which the experiment was received seem to warrant the purchase of additional tubes and making it a regular experiment to be performed by under-graduates taking a course in exact electrical measurements.

[2] Unsteady.

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AN ATTEMPT AT AN ELECTROMAGNETIC EMISSION THEORY OF LIGHT.

BY JAKOB KUNZ.

THE principle of relativity gives a consistent explanation of the phenomena of aberration of light, of the experiments of Fizeau and Michelson-Morley, and of the increasing mass of the electron as function of the velocity. The new principle rejects the ether, in which according to the older theory light waves are propagated and in which the electric and the magnetic energies have their seat. We are concerned again with actions at a distance, without a medium, but with actions proceeding with the velocity of light.

The mathematical simplicity of the original principle of relativity was mainly due to the fact that it used a fundamental constant, the velocity of light c as an absolute constant, so that the Lorentz transformation can be applied to Maxwell's equations, which remain unchanged.

Recently A. Einstein[1] generalized the original principle and applying it to the field of gravity came to the conclusion that c must not be considered as a constant but as a function of the coördinates. If the conclusion of this investigation is confirmed by the experiment, then the original theory of relativity fails and if it is not confirmed, the theory of relativity will be beset with great difficulties. In either case it will only be an approximation to the physical reality.

If we consider the material bodies as completely separated but exerting forces on each other, then the action at a distance remains incomprehensible at all events; but if there is no medium, we should expect in accordance with the Newtonian theory of gravity an action at a distance with infinite velocity, and as a matter of fact we do not know whether gravity proceeds with finite or infinite velocity. If however in the theories of relativity it is assumed that the action proceeds with constant or variable finite velocity, then the phenomena become even more mysterious.

The principle of relativity, even in its simple original form, affects our

[1] A. Einstein, "Entwurf einer verallgemeinerten Relativitätstheorie," Zeitschrift für Mathematik und Physik, Band 62, p. 225, 1914.

notions of space and time. Time, once absolute, dwindles to a mere shadow. The simultaneity of two events and the equality of two time intervals become relative, the parallelogram of velocities appears only as an approximation, an absolutely solid body is impossible and the mass of a body depends on its velocity.

When a physical theory which is mathematically complicated and is only an approximation cuts so deeply in our fundamental notions, and renders the phenomena so incomprehensible, the freedom of advancing other theories, which, though more conservative, attempt to coördinate the various phenomena in question should be granted. In the following a theory will be developed which agrees with that of relativity in many features, but gives an entirely different aspect of the world.

§ 1. Fundamental Assumptions.

1. One of the theories other than that of relativity is the electromagnetic emission theory of light. It is a compromise between the emission theory and the wave theory. Each electric charge is supposed to be surrounded by an electromagnetic field residing in the medium, which field itself forms the mass of the charge. Thus instead of having a continuous medium, ether, we have as many media as there are electric charges. Each individual electromagnetic field extends throughout the universe, but is essentially concentrated in the immediate neighborhood of the electron. No assumption is made as to the structure of the elementary medium.

2. Maxwell's equations will be applied to the molecular fields. The expressions for the masses of fields at rest will be extended to fields in motion.

3. The velocity of light is always equal to c for a vacuum. While in a mechanical emission theory the velocity v of the source is added geometrically to the velocity c, we have in the present theory, through a process of compensation, the velocity of light always equal to c, and independent of the velocity of the source. The difference between a mechanical emission theory, the undulatory theory and the electromagnetic emission theory of light can be illustrated by the following figures.

The source of light moves with the velocity v per second from A to B towards the observer. In the mechanical emission theory the light particles emitted in the point A with the velocity c would lie after a second on the sphere with radius c and with the center B. Thus the center of the wave front would always coincide with the source itself. In the undulatory theory, where the light is carried through the continuous

independent medium, the center of the disturbance would always coincide with the point A in which it has been emitted. In the electromagnetic emission theory the center of the disturbance would coincide with the moving source but the wave surface would be an ellipsoid of revolution whose equatorial plane is perpendicular to the direction of

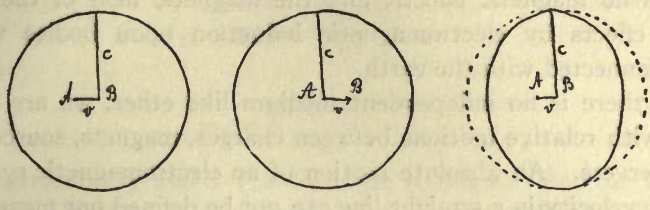

Fig. 1. Fig. 2. Fig. 3.

motion. In the second and third cases the velocity of light is always equal to c. In the second case the motion of the material luminous source has an influence on the optical phenomena, so we could hope to discover the motion of the source with respect to the ether and if the ether were at rest we could hope to discover the absolute motion of the source. This is impossible by mechanical methods according to New-

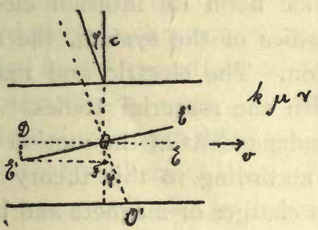

Fig. 4.

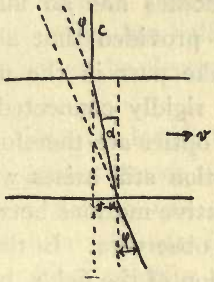

Fig. 5.

ton's principle of relativity. In the first and third theory we could not discover the absolute motion of the source. The critical velocity c of light in the vacuum is in Maxwell's theory equal to the ratio of the electrostatic to the electromagnetic unit of electric charge. If we consider c as constant and maintain Maxwell's equations unchanged for an electromagnetic field in motion, we consider that ratio of the two units also as independent of the motion. This means that the ratio of the force which on the one hand unit charge exerts upon another charge in a given distance to the force which on the other hand the same unit charge, when in motion exerts upon a magnet is independent of a uniform motion of the whole system. It is sufficient, but not necessary, for this purpose to assume that an electric charge

exerts upon another charge the same force, no matter whether both are at rest or in uniform motion; further, that an electric current exerts the same influence upon a magnet independent of the state of rest or of uniform motion of the whole system. In this way may be explained the facts that the electrostatic field of the earth, revolving round the sun, produces no magnetic effects, and the magnetic field of the earth no electric effects by electromagnetic induction upon bodies which are rigidly connected with the earth.

4. As there is no independent medium like ether, we are only concerned with relative motions between charges, magnets, sources of light and observers. An absolute motion of an electromagnetic system with constant velocity in a straight line can not be defined nor measured with optical and electrical methods.

The third and fourth assumptions lead to the Lorentz transformation of Maxwell's equations. There is however another transformation carried out by Maxwell and Hertz who found that the essential form of the equations remains unchanged if they are related to a system of axes at rest with respect to the ether or in motion similar to that of a rigid body; in other words, the absolute translation or rotation of a rigid system of bodies has no influence upon its internal electromagnetic phenomena, provided that all bodies of the system, the atomic fields included, take part in the motion. The electric and magnetic fields seem to be rigidly connected with the material bodies. The laws of geometrical optics are therefore independent of the motion of the earth.

The question still arises why according to this theory we can only discover relative motions between charges or magnets and between light sources and observers. In the first examples of course the reason lies in the interaction of the fields, but why should the field around a source of light contract in the equatorial plane if it approaches an observer? The reason may lie in the pressure which the light exerts upon the observer and which the observer exerts on the source. It might finally be possible that all the fields with which we can carry our experiments are imbedded as it were in a universal field of force.

§ 2. The Mass of the Electron.

An electron moves slowly in a medium whose permeability and dielectric constant are equal to unity. It is accompanied by a material electric field which, for small velocities, is symmetrical round about the spherical electron so that in a distance v from the center the electric force E is equal to $E = e/r^2$ and the magnetic force is equal to $H = ev \sin \vartheta/r^2 = Ev \sin \vartheta$; the magnetic energy per unit volume is equal to

$$E_1 = \frac{\mu H^2}{8\pi} = \frac{\mu E^2 v^2 \sin^2 \vartheta}{8\pi} = \frac{1}{2} m_1 v^2.$$

m_1 is the mass per unit volume.

$$m_1 = E^2 n' n^2 / 4\pi = \frac{\mu E^2 \sin^2 \vartheta}{4\pi}$$

or for $\mu = 1$

$$m_1 = \frac{E^2 \sin^2 \vartheta}{4\pi}$$

in general a

$$m_1 = \frac{\mu k^2 E^2 \sin^2 \vartheta}{4\pi}.$$

μ is the permeability and k the dielectric constant. For the following considerations it will be sufficient to put μ and k equal to 1. The mass dm of an infinitesimal ring will be equal to:

$$dm = \frac{e^2}{4\pi r^4} \sin^2 \vartheta \, 2\pi r^2 dr \sin \vartheta \, d\vartheta$$

and the whole mass will be equal to:

$$m = \frac{2}{3} \frac{e2}{a} = m_0,$$

where a is the radius, e the charge of the electron. This mass extends for an isolated electron throughout the whole space, but half of the mass is concentrated in the immediate neighborhood of the electron, that is in a sphere whose radius $a_1 = 2a$.

If the electron moves with finite velocity, then the electric field changes in such a way that the lines of electric force rotate towards the equatorial plane, which is perpendicular to the direction of motion v. At the same time the lines of magnetic force accumulate more and more in that plane as the velocity v increases. If finally the critical velocity c is reached, the whole electromagnetic field will be concentrated in that plane and the mass of the electron will increase indefinitely, so that an electric charge can not move with a velocity greater than that of light. We see also that in this limiting case the electron must cease to emit light in the direction of motion.

For a velocity v smaller than c we have:

$$E = \frac{e\left(1 - \frac{v^2}{c^2}\right)}{r^2\left(1 - \frac{v^2}{c^2}\sin^2\vartheta\right)^{\frac{3}{2}}},$$

$$dm = \frac{e^2\left(1 - \frac{v^2}{c^2}\right)^2 \sin^2 \vartheta}{4\pi r^4 \left(1 - \frac{v^2}{c^2}\sin^2\vartheta\right)^3} 2\pi r^2 dr \sin \vartheta \, d\vartheta,$$

whence

$$m = \frac{1}{2}e^2\left(1 - \frac{v^2}{c^2}\right)^2 \int\int\int \frac{dr}{r^2} \frac{\sin^3 \vartheta \, d\vartheta}{\left(1 - \frac{v^2}{c^2}\sin^2 \vartheta\right)^3}.$$

The integrations are to be extended over the whole field outside the electron. We do not know the shape of the electron, be it at rest or in motion. But there is a tension in the direction of electrical lines of force, and hence a resultant tension acting on the electron, especially round about the equator and the electron will assume the shape of an ellipsoid of revolution. According to the law which governs the equilibrium between internal and external forces, the mass as function of the velocity will be different. The integration will be carried out for three different conditions as follows:

1. The electron preserves the shape of a sphere during the motion.[1] The result of the integration is this:

$$\frac{m}{m_0} = \frac{3}{16}\left\{\frac{c^2 - v^2}{v^2} + 3 + \frac{c^2}{v^2}\left[\frac{3v}{(c^2 - v^2)^{\frac{1}{2}}} - \frac{(c^2 - v^2)^{\frac{1}{2}}}{v}\right] \operatorname{arctg} \frac{v}{\sqrt{c^2 - v^2}}\right\}.$$

2. The form of the electron changes according to the law

$$\frac{a}{b} = \sqrt{1 - \frac{v^2}{c^2}},$$

the integration yields the result

$$\frac{m}{m_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

the expression which relativity gives for the transversal mass of the electron.

3. The electron changes according to:

$$\frac{b}{a} = 1 - \frac{v^2}{c^2};$$

the integration gives

$$\frac{m}{m_0} = \frac{3}{8}\frac{c}{v}\frac{c^2 - v^2}{v^2}\left(\frac{c^2}{c^2 - v^2} - \frac{3}{4}\right) \log \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} + \frac{3c^4\left[2 - 3\left(1 - \frac{v^2}{c^2}\right)\right]}{16(c^2 - v^2)v^2}.$$

The first formula gives results which are smaller by 1 . . . 3 per cent. than the experimental values of C. E. Guye and S. Ratnowsky, which are however a little larger than those calculated by means of Abraham's

[1] J. Kunz, "Détermination théorique de la variation de la masse de l'électron en fonction de la vitesse," Archives des sciences physiques et naturelles de Genève, 1913.

formula. The third formula gives values too large and increasing too rapidly, while the second formula corresponding to relativity is in best agreement with the facts observed.

§ 3. The Electromagnetic Momentum and the Pressure of a Beam of Light.

It follows from Maxwell's equations that there is a tension in the direction of the lines of force, which per unit area perpendicular to the line is equal to the density of the energy. The pressure perpendicular to the lines of force is just as large. It follows that the pressure of a beam of light per unit area is equal to the electromagnetic energy per unit volume. We can now determine this pressure by means of the electromagnetic mass and momentum. A beam of light consists in the present theory of oscillating and advancing electromagnetic mass. The electric force is perpendicular to the direction of propagation, $\sin \vartheta = 1$ and if $\mu = k = 1$, then

$$m_1 = \frac{E^2}{4\pi}.$$

The momentum per unit volume is equal to

$$M = m_1 c,$$

the energy per unit volume will be

$$\frac{1}{2} m_1 c^2 = \frac{E^2 c^2}{8\pi} = \frac{H^2}{8\pi}$$

and the pressure per unit area is equal to

$$p = Mc = \frac{H^2}{4\pi}.$$

If

$$H = H_a \cos \frac{2\pi}{T}\left(t - \frac{x}{c}\right)$$

then

$$\overline{H}^2 = \frac{1}{2} H_a^2$$

and

$$p = Mc = \frac{1}{8\pi} H_a^2;$$

this is the energy of the beam per unit volume.

If k and μ are both equal to 1, then

$$p = m_1 c^2 = E, \quad m = m_1 = \frac{E}{c^2},$$

or by differentiation

$$dm = \frac{dE}{c^2}.$$

Hence it follows that a source of radiation, which emits energy, loses a part of its electromagnetic mass. The sun loses yearly about 10^{14} tons of electromagnetic inertia. On the other hand if a body absorbs energy, its mass must increase proportionally to the energy absorbed, and if an electric charge is set in motion, it will have more magnetic energy than at rest. If this electromagnetic mass were granular and could be broken up into smaller units, such as $E = hn$, then such a unit would have the mass for yellow light $m_1 = 3_1F.10^{-33}$, about 100,000 times smaller than the mass of the electron at rest.

§ 4. On Newton's Dynamical Equations.

Every atom possesses at least one electron. If the velocity of an atom changes, the inertia will change also. Newton's dynamical equations require therefore a correction which for all ordinary velocities of material ponderable bodies is insignificant, but which becomes very large, if the velocity v approaches that of light. The law of conservation of mass does not hold rigorously, but the law of conservation of momentum remains exact.

The total momentum remains constant in an enclosed system of heavy bodies, electrical charges, magnets, currents and sources of light. If a source emits a beam in a definite direction, it will lose momentum and be driven in the opposite direction. If on the other hand an electric wave strikes a charge, or if a beam is absorbed by a surface, then the material bodies gain as much momentum as disappears from the space. A force is defined in Newton's dynamics by the following equation:

$$F = \frac{dM}{dt},$$

but since

$$\frac{dM}{dt} = \frac{d(mv)}{dt},$$

$$F = \frac{mdv}{dt} + \frac{vdm}{dt}$$

and

$$Fdt = m\,dv + v\,dm.$$

If the mass moves through space dl during time dt, then the increase of energy is equal to:

$$dE = F\,dl = F\frac{dl}{dt}dt = Fvdt = dmv^2 + mvdv = c^2dm,$$

$$dm(c^2 - v^2) = mv\,dv,$$

$$dm\left(1 - \frac{v^2}{c^2}\right) = \frac{m}{c^2}v\,dv.$$

This equation has been integrated by Lewis and Tolman. Putting $v/c = x$, we get:

$$\frac{dm}{m} = -\frac{1}{2}\frac{d(1-x^2)}{1-x^2},$$

$$\log m = \log(1-x^2)^{-\frac{1}{2}} + \log m_0,$$

$$\frac{m}{m_0} = \frac{1}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}};$$

hence we find again for the increase of the mass the expression given by relativity.

The corrected equation of Newton holds not only for the ordinary inert bodies, but also for the radiations in a cavity. In a cavity bounded by perfect mirrors, we may find for the radiant energy E, the expression

$$E = mc^2,$$

or the energy of radiation possesses inertia. If moreover this electromagnetic inertia is subject to gravity, then the weight of such a cavity will be equal to:

$$mg = \frac{Eg}{c^2}.$$

If further the electromagnetic mass is at the same time heavy, then the gravity of the earth will exert on a certain body a force in a given point, which depends on the state of motion or rest of the body. An ordinary potential of gravity, as a function of the coördinates, only exists no more, for it now depends on the velocity of the falling body as well.

If the electromagnetic mass is subject to gravity, then a beam of light from a fixed star, passing through the field of attraction of the sun, will be attracted and therefore the position of the star will appear displaced. This very important problem may be solved by this phenomenon or also by observations made with pendulums of radioactive substances which are very rich in electrons. Let us consider two geometrically similar pendulums, the first consisting of a radioactive substance, such as radium, the second of non-radioactive substance. We shall assume the weight Mg of the two pendulums to be the same, but the mass M of the radioactive substance to be $m_1 + m$, where m_1 shall be subject to gravity, the electromagnetic mass m independent of gravity. The periods of the two pendulums will be

$$T_1 = 2\pi\frac{\sqrt{(m_1+m)r^2}}{Mgs},$$

$$T_2 = 2\pi\frac{\sqrt{m_1 r^2}}{Mgs}.$$

The radioactive pendulum would have a longer period than the ordinary one. 1 gr. radium contains about 1/13 mgr. more mass in the active state than after the transformations. In recent years it has been shown by Eötvös that for ordinary bodies the inertia is exactly proportional to the weight up to 10^{-7}. But nevertheless we have not yet a direct experimental proof that the electromagnetic mass is subject to gravity.

§ 5. The Experiment of Michelson-Morley.

As there is no independent medium, the motion of the earth has no influence upon geometrical optics and the result will remain the same whether we place the interference apparatus of Michelson-Morley in the direction of the motion of the earth or perpendicular to it. Even if the source of light were not connected with the apparatus, but were in motion, as for instance if the light of canal rays were made use of or the light of a star, in no case would we observe a displacement of the interference fringes through a rotation of the apparatus. Here appears a distinct difference between the electromagnetic and the mechanical emission theories. According to the latter theory we should expect an effect in the experiment of Michelson-Morley, if the light were incident from a star.

§ 6. The Experiment of Tronton and Noble.

The energy of an electric condenser of two plane parallel plates is independent of the direction of the motion of the earth; this experimental fact follows immediately from our assumptions. In the theory of an independent ether however the condenser would possess more energy if the plates were parallel to the velocity v of the earth, than if they were perpendicular to it. A charged and suspended condenser would produce a couple in the first position tending to bring it into the second position.

§ 7. Aberration of the Light from Fixed Stars.

While the light of a fixed star travels from the objective A of the telescope to O', the earth moves with the velocity v from O to O'. The phenomenon of aberration was always evidence in favor of an emission theory or led to the assumption of a stationary ether, through which the earth moves.

$$\frac{OO'}{O'A} = \frac{v}{c} = \frac{\sin\Phi}{\sin\vartheta} = 20''49.$$

Φ is the angle of aberration, v/c the constant of aberration of the light from fixed stars.

§ 8. The Experiments of Airy and Fizeau.

As the constant of aberration v/c depends only on v and c, Airy thought that it must change, if c changes. He filled therefore the telescope with water and expected a different angle of aberration, as the velocity of light in water is equal to c/r, if r is the index of refraction of water. Airy found however no change of the constant of aberration and he concluded that the water carries the ether with it, so that the velocity v is diminished by the same measure as c. If the water were carrying the ether with it with its own velocity, then no aberration would be possible, it must therefore communicate to the ether only a fraction of its own velocity. If the oscillating and advancing mass of a beam of light falls upon a transparent substance containing bound electrons, these charges will be set in motion and emit electromagnetic mass themselves. If moreover the substance struck by light is in motion, the electrons will be deviated from their original direction and oscillate in a new path. The light emitted will be perpendicular to this new direction and the original beam of light appears to be deflected from the original direction.

A beam of light strikes a column of water with plane surfaces, which move with constant velocity v perpendicular to the beam of light. Let us consider in a given point O of Fig. 4 an electron, which, if the water is at rest, under the action of the electric force OE, is deflected in the direction OD. The magnetic force would have no influence. If however the electron together with the water is set in motion with the velocity v, then the magnetic field of the light will act upon the charge in motion tending to deflect it in the direction OF. The resulting deflection and oscillation will be along OE'; the new beam will travel in a path perpendicular to this direction, that is from O to O'.

$$OD = eE,$$

$$OF = \frac{evH}{c^2} \quad H = cE,$$

$$OF = \frac{evE}{c},$$

$$\sin \Phi = \frac{OF}{OD} = \frac{v}{c};$$

this means that the angle of aberration Φ is independent of the specific properties k and r of the medium. Hence Fizeau's experiment follows immediately.

The water communicates to the beam a part of its own velocity v, so that the beam travels in the direction of v with a velocity u. It will

strike a point A on the lower side of the layer of water, and be deflected, so that Φ represents again the angle of deviation between the real and the observed beam. Now we have

$$\sin\alpha = \frac{v-u}{V},$$

$$\frac{\sin\Phi}{\sin\alpha} = r, \quad c = Vr,$$

$$\sin\Phi = r\sin\alpha = \frac{r(v-u)}{V} = \frac{r^2(v-u)}{c}.$$

This angle however is independent of the specific properties of the flowing substance; hence for the vacuum:

$$r = 1, \quad u = 0,$$

$$\sin\Phi = \frac{v}{c},$$

$$\frac{v}{c} = \frac{r^2}{c}(v-u), \quad (v-u)r^2 = v,$$

$$u = v\left(1 - \frac{1}{r^2}\right).$$

This is according to Fresnel and Fizeau the fraction of the motion, which the flowing water communicates to the beam of light.

If we observe a point at rest through a rotating disc of glass, it will appear to be deflected from its natural position. If we use a Roentgen ray instead of a beam of light then r the index of refraction is equal to 1 and therefore $u = 0$, that is, we would expect that Fizeau's experiment gives a negative result with Roentgen rays.

Recent Literature.

The present attempt at an electromagnetic emission theory is based upon the works of Faraday, Maxwell, H. A. Lorentz and other investigators. J. J. Thomson especially has in various investigations treated the electromagnetic field of an elementary charge as something individual, endowed with mass, momentum and energy. He has however, so far as I know, not extended the theory to the critical phenomena here treated. Contributions to the present theory have been made by N. R. Campbell in his book on modern electrical theory, by D. Comstock, Physical Review, 30, p. 267, 1910; R. C. Tolman, Physical Review, 31, p. 26, 1910; 35, p. 136, 1912; O. M. Stewart, Physical Review, 32, p. 418, 1911, and J. Kunz, American Journal of Science, 30, p. 313, 1910.

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SOME BRUSH DISCHARGE PHENOMENA PRODUCED BY CONTINUOUS POTENTIALS.

BY STANLEY P. FARWELL.

INTRODUCTION.

WHEN there exists a large difference of potential between a wire and neighboring conductors such as a similar and parallel wire, or a coaxial cylinder, the discharge phenomenon known as corona is likely to occur. For alternating differences of potential, this phenomenon has been extensively studied by Peek, Whitehead, Russell and others. The alternating corona takes the form of a more or less continuous and uniform bluish glow along the wire. The corona produced by continuous potentials has not received so much attention, presumably on account of the fact that its practical bearing on engineering problems is not so great. Watson[1] and Schaffers[2] have carried out experiments on the corona thus produced. Watson has experimented on wires as small as 0.7 mm. and at pressures as low as 360 mm. Schaffers has worked with cylindrical fields, using wires as fine as 0.006 mm. and various sizes of tube and has determined the critical voltage for visual corona at atmospheric pressure.

DESCRIPTION OF EXPERIMENTS.

The writer has been studying the corona as produced by continuous potentials for wires from 0.037 mm. to 1.285 mm. diameter and tubes 3.50 cm. and 4.45 cm. diameter. The relation between difference of potential and current between wire and tube has been studied for atmospheric pressure for the different sizes of wire; the critical voltage for visual corona has been obtained for pressures from somewhat above atmospheric down to 2.0 mm. of mercury and the character of the discharge noted; and the effect of variation of voltage for a constant low pressure has been investigated.

The object of this paper is to present especially some of the phenomena observed at these lower pressures, the influence of a short arc in series with the apparatus upon the character of the discharge, and the increase of pressure in the tube due to the ionization.

[1] Watson, Electrician, Sept. 3, 1909, Feb. 11, 1910, Feb. 18, 1910.

[2] Schaffers, Comptes Rendus, July, 1913, p. 203.

Influence of Pressure upon Character of Discharge.

The series of photographs in Fig. 1 represents the change in the discharge when the wire is negative to the tube and the pressure in the tube is varied from a low value up to atmospheric. The wire was No. 30 B. & S. (0.26 mm. diameter) bare copper taken from a coil obtained from the manufacturer. A glass tube 25 cm. long was lined with a brass sheath, except for a slit 6 mm. wide extending from end to end, and having an internal diameter of 3.5 cm. The wire was stretched tightly along the axis, passing through glass plates at the ends. The tube was rendered air tight by sealing wax and it could be exhausted through a branch tube attached to the side of the cylinder.

For the lowest pressures, the discharge takes the form of brilliant beads encircling the wire. Each has a bright cylindrical core, outside of which is a dark space which in turn is surrounded by a purplish glow extending out for some distance from the wire. As the pressure is increased, the difference of potential required to produce a discharge increases, and the number of beads increases. The central nucleus contracts and the bead becomes more and more like a brush until finally there is a line of brushes along the wire. Each brush consists of a bright spot on one side of the wire, with a fan-like purple glow spreading out from it, the plane of the fan being perpendicular to the axis. With the slit tube, the brushes point in different directions but if a similar test be run on a tube without a slit and one looks along the wire, the bright nuclei are seen to lie all in a plane, with alternate brushes on opposite sides of the wire, as a general rule.

As the pressure is raised toward atmospheric the isolated brush type of discharge gives place to such a discharge as is pictured in Fig. 1 for 357.0 mm. pressure. An occasional brush is left, mixed up with a more or less continuous glow which is very irregular. For atmospheric pressure, the discharge looks like the upper picture. The isolated brushes are very few, the rest of the wire presents an extremely "messy" appearance, the glow is bright and purplish and the discharge seems in constant movement.

For the lower pressures, a slight increase of voltage above that required to produce beads is sufficient to produce a violet arc-like discharge across the gap between wire and tube at one or two points and if this discharge be allowed to continue, the wire will be burned in two.

The photographs for 261.8 mm. pressure in Fig. 2 show the transition from one form of discharge to another, as it takes place for somewhat higher pressures. At the critical voltage, a continuous glow appears. Then as the voltage is raised slightly, the glow becomes spotted, followed,

at a higher voltage, by the gradual breaking up of the glow into the isolated brush form of discharge. Sometimes this process is not so gradual as here indicated. Suppose a difference of potential be impressed of a value above that required to just produce a glow. At the instant of closing the circuit, one sees a continuous glow which dissolves into the brush discharge, the brushes emerging one by one, until the entire wire is strung with them. The upper picture illustrates the regularity of spacing of the brushes, which will be taken up later.

The characteristic appearance of the discharge with the wire positive is that of continuous, uniform, bluish glow of diameter little greater than that of the wire. Its appearance is not noticeably changed by changes in pressure, but it gets brighter with increasing difference of potential.

Effect upon Discharge of an Arc in Series.

The effect upon the discharge of a short arc in series with the apparatus is shown in Fig. 2 for a pressure of 112.6 mm. When the wire is positive, the introduction of an arc causes the glow to brighten, increase in diameter, become more purple, and more ill defined as to boundary. The currents recorded on the photographs are obtained from the deflections of a D'Arsonval galvanometer. When the arc is introduced, the current so obtained is much less than one would expect from the small increase in resistance of the circuit caused by the arc. Evidently the discharge with arc in series is made up of two forms of discharge superimposed; the effect due to the continuous potential and an alternating effect caused by the oscillations set up in the circuit by the arc.

This superposition of effects is clearly illustrated when the wire is negative. The arc here causes a marked change in the discharge. The result is a continuous glow with a few isolated brushes strewn along it.

To test out the effect produced by the arc in apparently producing oscillations in the circuit, a condenser was connected across the cylindrical field. The introduction of the condenser caused the discharge to take the same form it had before the arc was introduced, except for there being a few less brushes. When there is a condenser thus in the circuit and the switch is closed, the transition from a continuous negative glow to the brush form of discharge is prolonged. With the condenser still in the circuit, the disconnection of the impressed difference of potential gives an opportunity for a discharge of the condenser across the cylindrical field. At the instant the line circuit is opened, no change in the appearance of the brushes is noticeable. Then as the condenser discharges and its potential falls, there is presented a "moving picture" of the stages of the discharge down to darkness. This discharge was a matter of several

seconds. As the voltage fell, the brush type of discharge was maintained: each regular arrangement of brushes giving place to another regular arrangement of fewer brushes. Since the resistance of the field is large, the condenser discharge must be of the continuous type.

Discharge between Parallel Wires.

Two No. 34 copper wires were placed parallel and 2 cm. apart inside a tube of glass 25 cm. long and the photographs of Fig. 3 were taken for pressures less than atmospheric. The tendency of the negative wire to show an isolated brush discharge and the positive to give a continuous glow is evident here. There is evidently a tendency for the positive sections of continuous glow to break up into spots or streamers. For constant pressure, the increase of the number of sections of the discharge with increase of voltage will be noted. The spacing of the sections is approximately regular and would undoubtedly be more so if the wires were more exactly parallel and stretched more tightly to make them straighter.

The two upper photographs show the effect produced by an arc in series. There is no longer the great difference between the appearance of the two wires. The negative wire, however, still shows a tendency to discontinuity of discharge. When the current is sufficiently great, violet streamers cross between the wires as shown in the upper picture. The current indicated is, again, only the component given by the galvanometer.

Spacing of Brushes as a Function of the Voltage.

The slit tube previously described was fitted with an arrangement for stretching the wire tighter and a series of photographs was taken of the discharge under constant pressure, with the wire negative and varying difference of potential. This series is shown in Fig. 4. The lowest picture shows the appearance of the discharge at a voltage little higher than that required to produce visual corona. It will be noticed that there are many tiny brushes and no regularity of spacing. For a little higher voltage, the number of small brushes has decreased and there are a number of large brushes disposed at quite regular intervals. The succeeding photographs show the effect of increasing the voltage still further. The number of brushes continually increases and the spacing is very regular. For the lower voltages, the brushes are fixed in position for a given voltage and will always show up in the same position as the circuit is interrupted and then closed again. When the voltage approaches the value at which there will be an arc between wire and tube, each brush is in constant movement back and forth in a short path, but the number of brushes is constant for a given voltage.

Fig. 5 shows the relation between difference of potential and current. In connection with this graph it might be noted that the critical voltage for visual corona was 2,440.

It would appear from a close observation of the character and spacing of the brushes that there are only certain voltages for which there appears a regular distribution of full-sized brushes. For intermediate voltages,

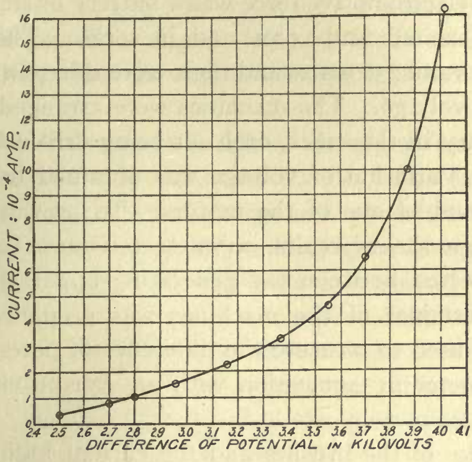

Fig. 5.

there is more or less irregularity in the size of the brushes and their spacing. For the pictures of Fig. 4 an effort was made to pick out those points at which the distribution was the most regular. Fig. 6 shows the variation with the difference of potential of the number of full-sized brushes and the current per brush.

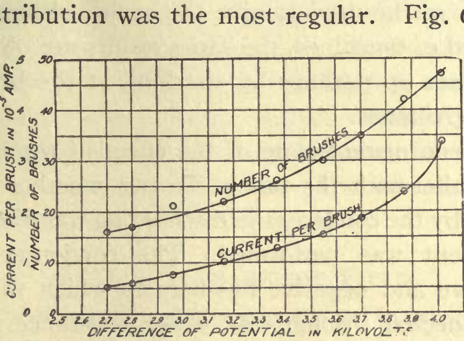

Fig. 6.

When there was a marked variation in the size of the brushes an estimate was made of the equivalent number of full-sized brushes. These graphs clearly indicate that a definite relation exists between the voltage and the number of brushes, for a given pressure, and that the current per brush is not a constant but also varies with the voltage.

The question may be raised as to whether the isolated brush form of discharge may not be due to oscillations in the circuit. In order to make it clear that this is essentially a direct current phenomenon, there is given below a description of the generating apparatus used in producing the continuous potentials and some experiments and arguments which support this view.

The source of electromotive force was a battery of small direct-current generators, self-excited and connected in series. The machines were rated at 500 volts and 250 watts and there were thirty in all, giving 15,000 volts at normal voltage. The machines were arranged in two sets, ten in one and twenty in the other, each set being driven by its own direct current motor. Variation of voltage was obtained by field control of the generators and of one of the motors. To prevent damage to the machines through short-circuits, a water resistance of a rather large value was connected between the generating apparatus and the tube. The negative terminal of the machines was grounded. Electrostatic voltmeters were used to measure the difference of potential and a D'Arsonval galvanometer in connection with an Ayrton shunt box to give the current.

The appearance of the brushes and the current indicated by the galvanometer are constant for a given voltage, no matter what combination of machines are used as the source of potential. One of the sets may be used and the appearance of the spots and the voltage and current noted. Then if the other set be used to give the same voltage with a different number and speed of machines, the same results are obtained. If there were oscillations set up perhaps by sparking at the brushes, we would not expect this agreement.

Mention has been made before of the effect of the introduction of a condenser in parallel with the tube. To test whether the current sent through the tube by the condenser in discharging was direct or oscillatory, another experiment was performed. The condenser was connected across the positive and negative bus-bars to which the generating apparatus was connected through the water resistance. Then a switch connecting the machines to the bus-bars was closed as was also a switch leading to the tube. The deflection of the galvanometer was noted and the appearance of the brushes. Then the generator switch was opened and the condenser discharged through the tube and the galvanometer. After the switch was opened, the galvanometer deflection gradually decreased, the rate of decrease of the deflection being slower and slower as the discharge proceeded. The opening of the switch caused no immediate change in the brushes, only the gradual change already noted.

That the discharge of the condenser must be continuous is shown by the deflection of the galvanometer and it can be further proved by a rough calculation. Assuming the resistance of the cylindrical field as given by E/I and taking a set of values of E and I for the comparatively low pressures at which the brushes are best formed, we obtain $R = 1.83 \times 10$ ohms. Assuming the very large value of 0.1 henry for the inductance of the circuit, and the approximate value of 2 mf. for the capacity, we find the R is about 4.1×10^4 times as great as $\sqrt{4L/C}$ and hence it is clear that the condenser discharge must be of the continuous type.

By running wires from the terminals of an induction coil to the central wire and the tube and then adjusting the discharge points on the coil to such a distance that a silent discharge took place between them, it was possible to obtain an almost uniform hazy glow along the wire. But no effect could be obtained like the uniformly spaced brush discharge.

It is well known that an arc is the source of electrical oscillations and it has been shown by a previous figure that a short arc in series with the tube disturbs the brushes due to the direct current by the superposition of an alternating current effect so that the glow becomes more or less uniform and the difference in the appearance of the glow for different polarities becomes much less. So the introduction of an oscillatory current acts to suppress the isolated brush form of discharge and not to cause it.

The difference between positive and negative electricity is hardly better demonstrated by any other phenomenon. It should be stated here, however, that Peek[1] by a stroboscopic method has also observed "more or less evenly spaced beads" on the negative wire when there was corona between parallel wires caused by an alternating difference of potential of 80,000 volts at atmospheric pressure. The wires used by Peek were 0.168 cm. in diameter, spaced 12.7 cm. apart.

Effect of Magnetic Field.

A strong horseshoe electromagnet was placed in various positions with its poles against the tube and the effect upon the various forms of discharge of making and breaking the magnet circuit was observed. No change could be noted in the appearance of the discharge or the current flowing.

Variation of Pressure in Tube with Voltage.

A No. 36 B. & S. copper wire, 0.135 mm. in diameter, was stretched tightly along the axis of a brass tube 4.45 cm. in diameter and closed at

[1] Proc. A. I. E. E., Vol. 31, No. 6, p. 1123 and Plate LXV.

the ends by glass plates through which the wire passed. A small branch tube was soldered to the side of the main tube and from it connection was made to an air-pump. An open manometer of small bore containing a light oil was connected to the side of the branch tube. Everything was rendered airtight after disconnecting the manometer and the tube was exhausted. Then dry air was gradually admitted through a tube containing soda-lime and a wash-bottle containing concentrated sulfuric acid, until the pressure was again atmospheric, 744.0 mm. in this case. The manometer was again connected and various differences of potential impressed.

As soon as the voltage reached the critical value to cause an appreciable current to flow, a jump in the columns of the manometer was apparent. This jump occurred lower for the wire negative and it was difficult to tell just the voltage at which it began. When the wire is negative, any little dust particle on it will be sufficient to start a discharge at a lower voltage than would be required to cause a general glow along the whole wire. But for the wire positive, the critical point is very marked and the jump occurs, as closely as one can judge, at the same time that a faint bluish glow is seen along the wire. Fig. 7 shows the increase in the

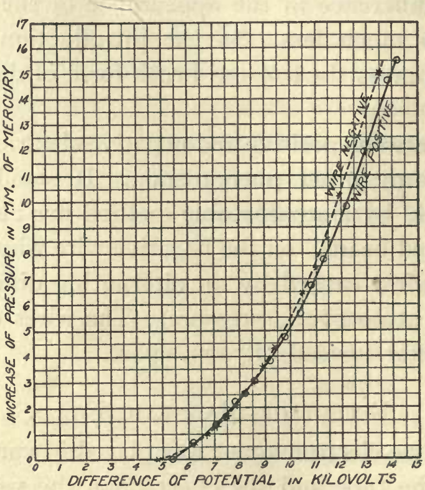

Fig. 7.

pressure as the voltage is raised. This graph has exactly the same appearance as the graph plotted between voltage and current, as one might expect from the theory of the conduction of electricity through gases. It will be noted how the curves for the two polarities cross at low voltages and that the increase of pressure for a given voltage is greatest for negative

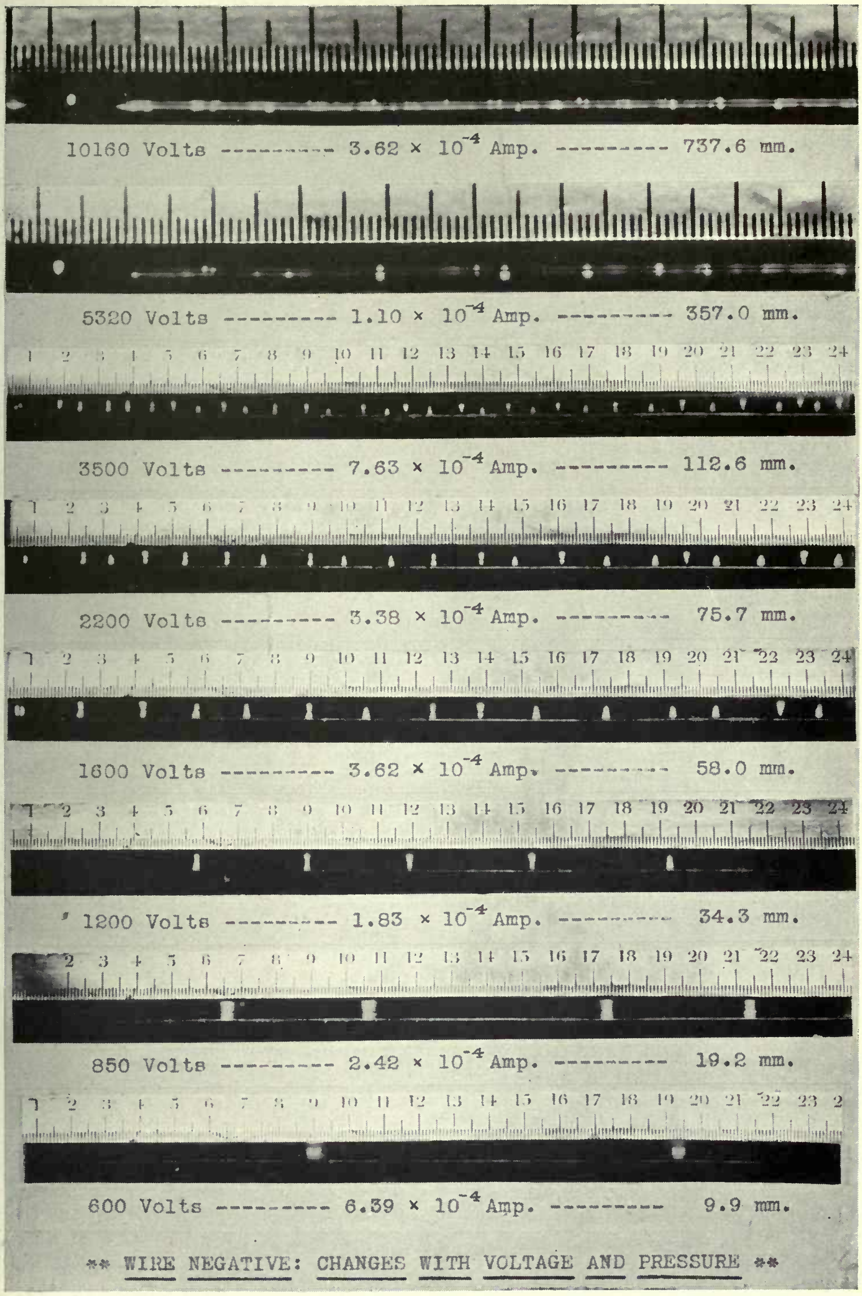

Fig. 1.

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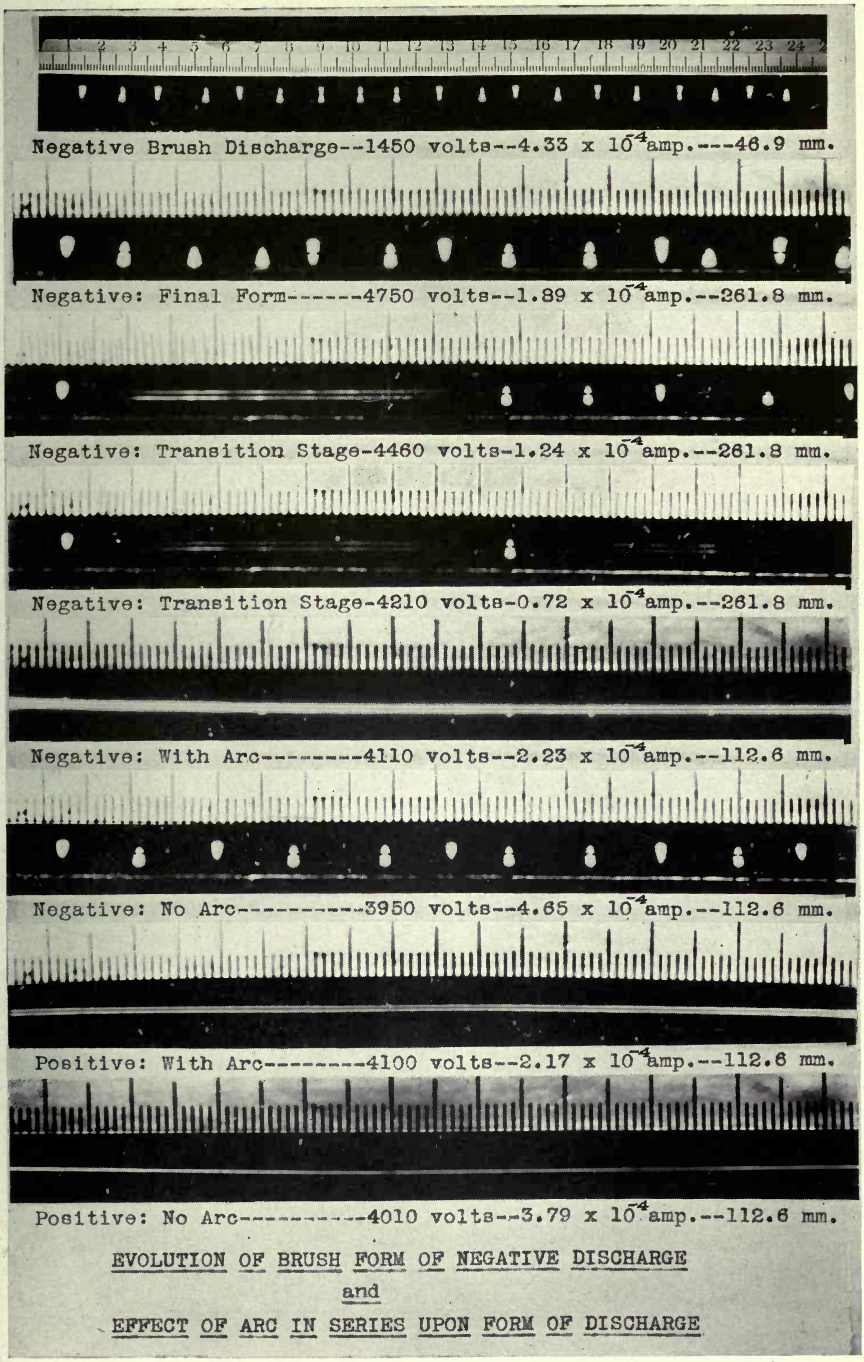

Fig. 2.

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Physical Review, Vol. IV., Second Series
July, 1914.

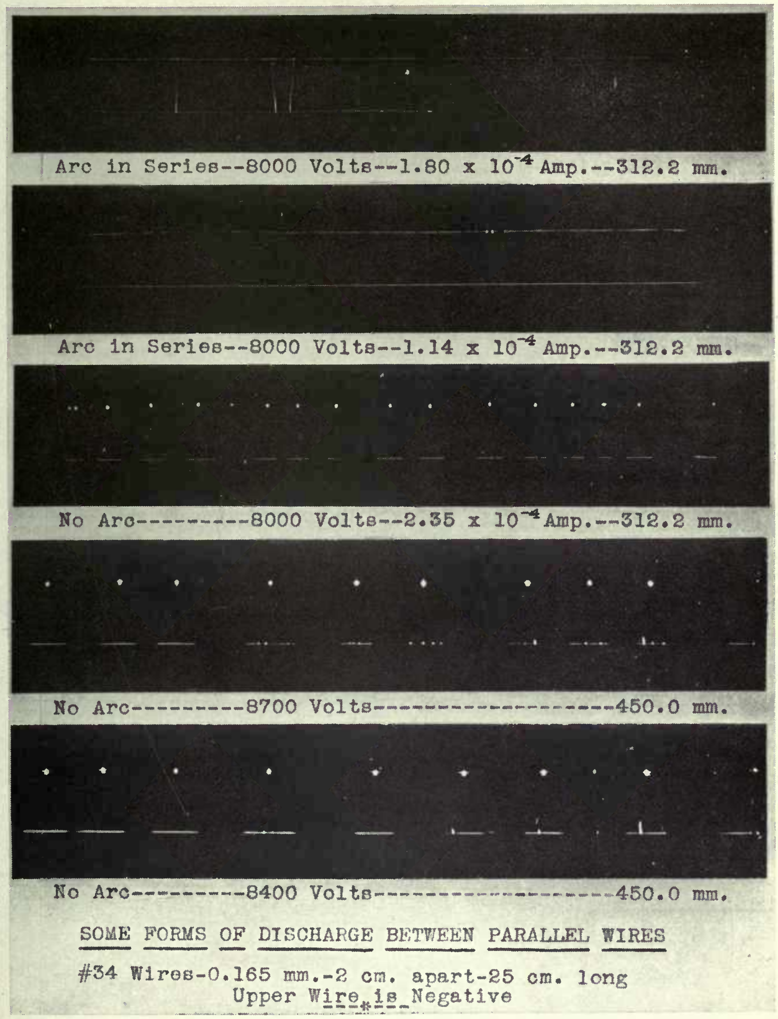

Fig. 3.

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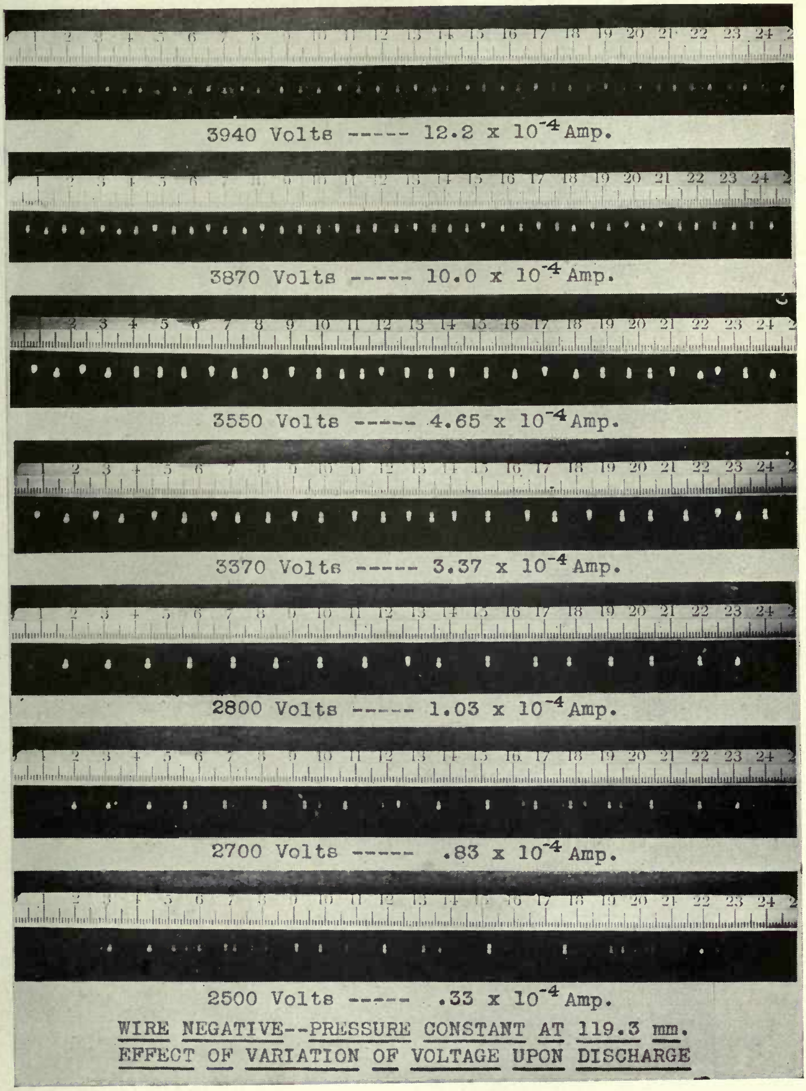

Fig. 4.

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polarity of the wire during the greater part of the range. The crossing of the curves is typical of the voltage-current graphs.

In addition to the sudden jump at closure of the circuit there is a gradual increase of pressure due to the heating effect of the current and hence care was taken to read the heights of the columns of liquid as soon as possible after the circuit was closed.

The work upon which this paper is based was performed in the laboratory of physics at the University of Illinois under the direction of Dr. Jacob Kunz, asst. prof. of physics. To him and to Prof. E. B. Paine, of the electrical engineering department, the writer wishes to acknowledge his indebtedness for many helpful suggestions as to the conduct of this work.

Laboratory of Physics,
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April, 1914.

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THE DIFFUSION OF GASES AT LOW PRESSURES MADE VISIBLE BY COLOR EFFECTS

AN interesting and instructive experiment for the lecture table is to connect a discharge tube *AC,* which is about one meter or more in length and which has the exhaust nipple at one end, to a pump that will give a Geissler vacuum—an oil Geryk pump will answer very well. Between the pump connection *M* and the valve *O* that closes the tube there should be fused a side branch *N* also having a valve. Connect *N* by a rubber tube to some source of gas other than air, *e. g.,* ordinary illuminating gas. The connection at *M* should be made direct to the pump. Connect *A* and *C* to the terminals of an induction coil that will give a spark in air five or more centimeters long.

To operate, close the valve in the branch *N,* open *O* and evacuate the discharge tube to the point where on sparking the characteristic striæ show distinctly. It is immaterial whether *A* or *C* is the cathode, or whether the discharge is unidirectional. Now close the valve *O,* and, with the pump still running, open *N* partly, allowing illuminating gas to be drawn by the pump through the branch *OM,* thus displacing the air by the gas. By closing *N,* pumping and later admitting more gas, every trace of air may be washed out of

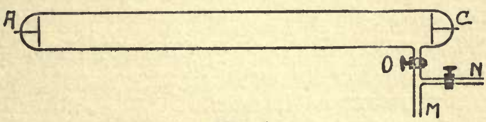

FIG. 1.

the tube leading up to *O*. Now with *N* closed allow the pump to run for a few seconds until it is judged that the pressure in the connect-

ing tube MO is about that in the discharge tube AC.

At this stage everything is in readiness for the experiment, namely, the diffusion of gases at low pressures made visible by the color effect. The well-known characteristic color of the discharge in the case of residual air, containing possibly some water vapor, is orange red. To now introduce the illuminating gas open the valve O for a moment, then close it. The end C of the discharge tube is instantly filled with a beautiful greenish-white color characteristic of illuminating gas. This color will diffuse slowly towards A, each color paling out, and after three or four minutes the discharge throughout the tube will assume a uniform grayish hue. The rate of diffusion is surprisingly slow and of course depends upon a number of factors, *e. g.*, the gas pressure in the tube, the pressure of the gas that is admitted, the ionization within the tube due to the discharge passing through the tube, the amount of moisture present, etc.

If now the gas connection at N be removed and this stem opened to the air the pump and connections may be freed of gas and the inverse experiment performed; namely, that of introducing a small quantity of air. The resulting orange red color and its diffusion through the grayish hue of the illuminating gas is even more striking than the first.

The success of the experiment depends largely upon the skill of the operator in properly proportioning the quantity of gas to be introduced. It is a very simple experiment to perform.

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ON THE PRESENT THEORY OF MAGNETISM.

BY JAKOB KUNZ.

THE electron theory seems to account for the magnetic phenomena in a very direct way. Indeed we have only to assume that the molecular currents of Ampere, which form the elementary magnets, are revolving electrons in order to express Ampere's theory of magnetism in terms of the electron theory. Nevertheless it was only on the basis of the researches of P. Curie that P. Langevin was able to account for the difference in diamagnetism and paramagnetism. Curie found that the diamagnetic susceptibility is independent of the temperature while the paramagnetic susceptibility is inversely proportional to the absolute temperature. Langevin concluded that there is a fundamental difference between diamagnetic and paramagnetic properties. In Langevin's theory the diamagnetism is a characteristic property of each atom which contains a certain number of revolving electrons. If the resultant magnetic moment of these electrons for an external point is zero, then the body is diamagnetic, the action of an external magnetic field consists in a change of the orbit, giving rise to the diamagnetic modification of the atom. If the revolving electrons possess a resultant magnetic moment, the body is paramagnetic. Matter in all its forms is diamagnetic; paramagnetism, whenever it appears, covers as it were the diamagnetism, and there is no transition between the two distinct groups. We shall add a short deduction of the diamagnetic susceptibility.

We consider in a diamagnetic gas an atom with an electronic orbit, of radius r, the electron e revolving with velocity v, in a plane perpendicular to the magnetic field. The moment will be equal to $M = \pi r^2(e/T)$ without magnetic field; if a field H is applied, the time of vibration T and the angular velocity will change, so that

$$dM = e\pi\left(\frac{2r}{T}dr - \frac{r^2}{T^2}dT\right),$$

or neglecting the first part

$$= - e\pi r^2 \frac{dT}{T^2},$$

$$\frac{mv^2}{r} = f.r \text{ or } f = \frac{mv^2}{r^2},$$

$$\frac{mv^2}{r'} = fr' - \text{Hev}; \frac{mv^2}{r'^2} = f - \frac{\text{Hev}}{r'} = \frac{mv^2}{r^2} - \frac{\text{Hev}}{r'},$$

$$\frac{mv^2}{r'^2} - \frac{mv^2}{r^2} = \frac{\text{Hev}}{r'},$$

$$m2\pi\left[\frac{1}{T'^2} - \frac{1}{T^2}\right] = \frac{He}{T'},$$

$$m4\pi\frac{dT}{T^2} = He; \frac{dT}{T^2} = \frac{He}{4\pi m},$$

$$dM = \frac{-e^2r^2}{4m}H.$$

If there are N orbits per unit volume and if the axes are uniformly distributed in all directions, then we have

$$M = -\frac{e^2r^2N}{12m}H, \text{ or } k = -\frac{e^2r^2N}{12m}.$$

Apparently N and r are independent of the temperature. This theory of Langevin of the diamagnetic susceptibility k is at the same time the theory of the Zeeman effect.

In order to find the expression for the paramagnetic susceptibility fo a gas, we shall use a method quite different from that of Langevin. Let the angle between an external magnetic field H and the direction of the moment M of an elementary magnet be equal to α, the work required in order to rotate the magnetic particle from the direction H into its present direction will be equal to $W = -MH\cos\alpha + C$; the heat of the gas will change by this amount, and in order to keep the temperature constant, we have to add a quantity of heat $Q = -W = MH\cos\alpha$, and the increase of entropy will be equal to $S = (Q/T) = (MH\cos\alpha/T)$; and this entropy will be proportional to the lograithm of the probability P that we find the magnet in the direction α

$$S = R\log P + \text{const.} = \frac{MH\cos\alpha}{T},$$

$$P = e\frac{MH\cos\alpha}{RT},$$

and the number of magnets which are found in an angular interval $d\alpha$ will be proportional to P, or

$$dn = Ke\frac{MH\cos\alpha}{RT}\cdot d\omega,$$

where

$$d\omega = 2\pi \sin \alpha \, d\alpha,$$

$$dn = Ke\frac{MH \cos \alpha}{RT} 2\pi \sin \alpha \, d\alpha,$$

the total number N of molecules per unit volume will be

$$N = 2\pi K \int_{\alpha}^{\pi} e^{a \cos \alpha} \sin \alpha \, d\alpha,$$

where we put a for (MH/RT),

$$N = \frac{4\pi K}{a} \sin ha$$

and the intensity of magnetization $\mathfrak{J}$ becomes:

$$\mathfrak{J} = \int_0^{\pi} M \cos \alpha \, dw$$

$$= MN\left(\frac{\cos ha}{\sin ha} - \frac{1}{a}\right).$$

The maximum value of the intensity of magnetization $\mathfrak{J}_m = MN$, hence

$$\mathfrak{J} = \mathfrak{J}_m\left(\frac{\cos ha}{\sin ha} - \frac{1}{a}\right) = \mathfrak{J}_m\left(\frac{1}{3}a - \frac{2}{90}a^3 + \frac{4}{45 \cdot 42}a^5 - \dots\right).$$

Neglecting the higher powers of $a = MH/RT$ we find

$$\mathfrak{J} = \mathfrak{J}_m \frac{MH}{3RT} = kH,$$

$$k = \frac{NM^2}{3RT},$$

that is, the paramagnetic susceptibility is inversely proportional to the absolute temperature; that is the rule of Curie.

Experimental Facts.

The phenomena are far more complicated than the theory of Langevin indicates. The investigations of H. DuBois, K. Honda, M. Owen, Kamerlingh Onnes, P. Weiss, A. Perrier and others have revealed a very large variety of phenomena, in which the rules of Curie are altogether exceptions so that we have to extend or abandon the present theories.

The diamagnetic susceptibility should be an atomic property, which is independent of temperature, of a change of state, of a polymorphic transformation or of chemical combination. This is not the case. For instance, the diamagnetic susceptibility of amorphous carbon, of Cu, Zn, Zr, Cd, $\mathfrak{J}$n, Sb, Te, Tl, $\mathfrak{J}$, Pb, Bi, decreases with increasing temperature, k in the melting of Ag, Sn, Sb, Ga, Ge, Au, Hg, Tl, Pb, Bi changes

discontinuously. In the polymorphic transformation of C, S, Sn, and Tl the susceptibility changes abruptly, even the sign changes in the polymorphic transformation and during the melting process of tin. In the case of boron (0–400° C.), diamond, silver and iodine (0–114°) the diamagnetic susceptibility increases with the temperature. There are only a few elements whose diamagnetic k remains constant within a certain interval of temperature; the diamagnetic susceptibility of an inorganic compound is not an additive property. Oxygen, for instance, is a strongly paramagnetic element, but if it combines with the paramagnetic elements of Be, Mg, Al, Mo, W, Th, it forms diamagnetic oxides. And in general the diamagnetic and paramagnetic properties depend so much on physical and chemical influences, that one might be inclined to ascribe them to electrons which are revolving on the surface of the atom. In organic compounds, at all events, it has been shown by P. Pascal that the molecular susceptibility X is an additive property of the atomic susceptibility. Oxygen in these compounds may be para- or diamagnetic. In more complicated compounds, the structure has a great influence on X. The diamagnetic constants are on the whole not smaller than the positive paramagnetic values.

The diamagnetic susceptibility of graphite is greater than the paramagnetic susceptibility of such an element as manganese, one of the strongest paramagnetic elements; charcoal, bismuth and antimony have also large negative susceptibilities. Besides in the crystals of graphite and antimony k varies with the direction of the axes. All these facts seem to indicate that what we observe is the difference between a positive and negative magnetism.

A similar variety of phenomena is observed in paramagnetism. Oxygen follows Curie's law at ordinary and at higher temperatures, but at lower temperatures the susceptibility varies inversely as the square root of the absolute temperature and finally probably becomes constant. Over certain intervals of temperature the susceptibility remains constant in the elements: Na, Al, K, V, Cr, Nb, W, Os, and even increases with increasing temperature in the case of Ti, V, Cr, Mn, Mo, Ru, Rh, Ir, Th. The ferromagnetic metals above the critical temperature, where the ferromagnetism disappears, seem to follow Curie's law, probably with the exceptions of the compounds Fe_3O_4 and pyrrhotite.

An extension of Langevin's theory is necessary both for the diamagnetic and the paramagnetic susceptibility. In the first place, in Langevin's theory it is silently assumed that the moments of the elementary magnets are independent of the temperature. This assumption is by no means self-evident. The electrons revolve in the outer layers

of the atoms probably and the moment of a molecule is the resultant of the moments of the atoms. With increasing temperature we have reasons to believe, the atoms share the energy of temperature agitation and the resultant moment of the molecule may be affected. Besides the fact that the diamagnetic susceptibility changes abruptly, in polymorphic transformations, in changes of state, in chemical transformations, indicates, that the diamagnetism is not simply an additive atomic property. In general we have to put:

$$M = M_0 f(T).$$

In the second place Langevin's theory of paramagnetism applies only to gases and dilute solutions. The resultant magnetic moment per unit volume depends only on the directing power of the field and on the "scattering" power of the temperature agitation. The equilibrium between these two effects leads to Curie's rule. But as soon as we consider a more condensed state of aggregation, the molecules will exert an influence on each other, and this influence for crystals will vary in different directions. P. Weiss has indeed extended Langevin's theory to ferromagnetic substances by adding to the external field H an internal or molecular field $N\mathfrak{J}$ which is proportional to the intensity $\mathfrak{J}$ of magnetization. In this way P. Weiss was able to explain a large number of phenomena of ferromagnetic crystals. A similar influence however must exist in paramagnetic solid and liquid substances. The mutual action of the molecules will be a certain function of the temperature $f(T)$ and will in general oppose the tendency of the external field to direct the elementary magnets, just as the temperature agitation; so that the energy of the opposing forces may be written in the form; $RT + f_1(T)$; the parameter a will now be equal to $\dfrac{M_0 f(T)H}{RT + f_1(T)}$ and $\mathfrak{J}$ will become:

$$\mathfrak{J} = M_0 f(T)N\left[\frac{\cos ha}{\sin ha} - \frac{1}{a}\right]$$

$$\mathfrak{J} = M_0 f(T)N\left[\frac{1}{3}a - \frac{2}{90}a^3 + \dots\right]$$

or approximately:

$$\mathfrak{J} = [M_0 \cdot f(T)]^2 \frac{HN}{3[RT + f_1(T)]},$$

$$k = \frac{[M_0 f(T)]^2 N}{3[RT + f_1(T)]}.$$

if $f(T)$ is a constant, for instance, equal to 1 and $f_1(T)$ also a constant ΘR, then we find

$$k = \frac{M_0^2 N}{3(RT + \Theta R)}$$

or

$$k(T + \Theta) = \text{constant},$$

a result which has been deduced by E. Oosterhuis[1] from Planck's theory of quanta of energy by entirely different considerations. At very low temperatures the specific heat of all substances seems to approach zero, the coefficient of expansion approaches zero also, the electrical conductivity of metals becomes very large, if not infinite, the thermal conductivity increases rapidly. All these properties can be explained by the assumption that at these very low temperatures in the neighborhood of the absolute zero the molecules gradually lose their mobility, and a given substance at absolute zero is a real solid body, as it were one large molecule, where the molecular mobility has disappeared; that means that the influence of the temperature agitation of the individual elementary magnets becomes weaker and weaker and that we can not even define the molecular magnet, because the whole system of magnets is as it were solidified, so that even at the absolute zero saturation of a substance is impossible and that the influence of temperature becomes smaller and smaller or the paramagnetic susceptibility becomes constant. This seems to be the tendency of solid oxygen.

If now at the lowest temperatures the function $([f(T)]^2/RT + f_1(T))$ is a constant, and at rather high temperatures is equal to $1/T$ and changes continuously from one extreme to the other, then it will in intermediate temperatures be approximately equal to $1/T^{\frac{1}{2}}$, and for this interval we shall have:

$$k = \frac{M_0^2 N}{3T^{\frac{1}{2}}},$$

a result which has been obtained by H. Kamerlingh Onnes and A. Perrier for liquid and solid oxygen. The formula $k(T + \Theta) = C$ or $X(T + \Theta) = C$, where X is the molecular susceptibility will be tested by means of measurements made in Leiden[2] on manganese sulfate.

In the next place the free electrons will be considered as contributing to the diamagnetic susceptibility. It has been shown by H. E. Dubois and K. Honda that the diamagnetic susceptibility of amorphous carbon, Cu, Zn, Cd, In, 𝔍, Sb, Te, Tb, Pb, Bi, Sb decreases with increasing temperatures. The strongest diamagnetic metals are Sb and Bi, which show also a very large Hall effect. The magnetic field seems to act on

[1] Die Abweichungen vom Curie'schen Gesetz im Zusammenhang mit der Nullpunktsenergie, Phys. Zeitschrift, Vol. XIV., p. 862, 1913.

[2] Leiden, Comm. No. 132e.

$M_nSO_4H_2O$				M_nSO_4			
Tabs.	$X \cdot 10^6$ Abs.	$C10^{10}$	$\Theta = 1.18$	Tabs.	$X \cdot 10^6$	$C10^{10}$	$\Theta = 26.5$
288.7	66.3	1.92		293.9	82.8	2.82	
169.6	111.5	1.905		169.6	144.2	2.83	
77.4	247	1.93		77.4	274.8	2.85	
70.5	270	1.93		64.9	314.5	2.87	
64.9	292	1.93		20.1	603	2.82	
20.1	914	1.94		17.8	627	2.78	
17.8	1,021	1.94		14.4	636	2.60	
14.4	1,233	1.92					

the free electrons so that they move in spirals or circles and produce diamagnetism. E. Schrödinger[1] found for the contribution k of the diamagnetism, made by the free electrons:

$$k = -\frac{1}{3}\frac{e^2}{m}\lambda^2 N,$$

where N is the number of free electrons per unit volume, and λ the mean free path. The effect, calculated in this way, is 100 times too large for silver and copper, which seems to be another argument against the "free" electrons in metals. Nevertheless the action of temperature on diamagnetism and the fact that Sb and Bi have a large Hall effect and a large negative magnetism indicate that the conduction electrons contribute somewhat to the diamagnetism.

The Periodic System of the Elements and Their Magnetic Properties.

The elements may be arranged in series according to the atomic weights in different ways. A certain periodicity between atomic weights and magnetic properties always appears. If the atomic weights are represented by abscissæ and the magnetic susceptibilities as ordinates, the curve obtained is of a most irregular character, representing seven distinct maxima, among which that of the iron group is by far predominating. If only the sign of the magnetic properties is taken into account, one gets the best representation perhaps by the method of the helix due to B. K. Emerson, which is given in Fig. 1.

The strongly magnetic groups appear on a diameter, where we find Fe, Ni, Co, then Pd, Ru, Rh, then Gd, En, Sm, then Pt, Ir, Os. Moving on the spiral from iron to the right, we meet Mn and Cr, elements which are paramagnetic, but whose strongly magnetic properties appear only in some of their alloys and compounds such as the Heusler alloys, manga-

[1] Kinetische Theorie des Magnetismus, Sitz. Ber. der K. Akademie der Wissenschaften, IIa, Bd CXXI., p. 1305, 1912.

nese-antimony, manganese-tin, manganese-zinc, Cr_5O_9. On the right hand side from the ferromagnetic elements, there are paramagnetic elements, on the left hand side the diamagnetic elements. Opposite to the magnetic metals there are the inert gases, which seem to be weakly diamagnetic. On the right-hand side of the inert gases we find the alkali metals whose weakly paramagnetic properties are not yet sufficiently known. The strongly magnetic metals, cobalt, nickel and iron, belong to the elements with minimum compressibility, with most complex

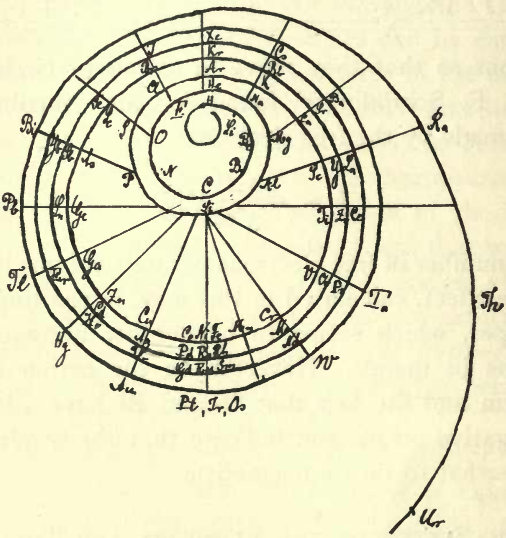

Fig. 1.

spectra, with complex double salts, with great condensation of mass, the heavy metals. Thus it looks as if condensation of electronic orbits were a maximum in these ferromagnetic metals and that the magnetic properties were related directly or indirectly to the mechanical, optical and chemical properties. It is very remarkable that immediately after the strongly magnetic metals there follow the diamagnetic metals:

	k
Cu	−0.66
Ag	−1.4
Tb	
Au	−2.6

On the next diameter we have:

Zn	−0.96
Cd	−1.16
Ho	
Hg	−2.6

When we move outward on a diameter of the spiral, the diamagnetic

susceptibility increases. The same rule is repeated by chlorine, bromine and iodine; sulphur, selenium and tellurium; phosphorus, arsenic, antimony, bismuth. If a represents the atomic weight, α and β two constants, then the atomic susceptibility can be represented for the last three groups by

$$X_a = - C_e^{+\beta a - \alpha}.$$

The same law seems to hold for all the groups of diamagnetic elements which are in the previous representation on the left of the diameter passing through the iron group and the inert gases. Thus, for instance, for zinc, cadmium and mercury we find:

$$X_{zn} = \frac{0.96 \cdot 65.4}{7.1} 10^{-6} = 8.83 \cdot 10^{-6},$$

$$X_{Cd} = \frac{1.16 \cdot 112.4}{8.6} 10^{-6} = 15.2 \cdot 10^{-6},$$

$$X_{Hg} = \frac{2.6 \cdot 200}{13.6} 10^{-6} = 38.3 \cdot 10^{-6}.$$

If we put

$$x_a = 10^{\beta . a + \alpha}$$

we find from Cd and Hg for α and β the values:

$$\alpha = 0.6146, \; \beta = 0.00502,$$

$\log X_{zn} = 1.583$, the calculated value $= 1.618$.

The agreement is not so good for the last diamagnetic group of elements: Cu, Ag and Au, here the atomic susceptibilities are as follows:

$$X_{cu} = 5.29 \cdot 10^{-6},$$

$$X_{Ag} = 14.39 \cdot 10^{-6},$$

$$X_{An} = 26.55 \cdot 10^{-6}.$$

The copper seems to make an exception. Whether this is due to an inaccurate determination of X or to the fact that this element follows immediately after the iron group, remains an open question. Between the Zn, Cd, Hg group and the P, As, Sb group there are two more groups of diamagnetic elements, namely, those of Ga, $\mathfrak{J}$n, Tl and Ge, Sn, Pb. The few values of X known for these elements show that this magnetic constant increases toward the periphery along the diamagnetic diameter of the spiral.

If we travel along the spiral from copper towards zinc and from silver toward cadmium, we find the following values for the atomic susceptibilities.

Cu	$5.29 \cdot 10^{-6}$	Ag	$14.4 \cdot 10^{-6}$
Zn	8.83	Cd	15.2
Ga		𝔍n	
Ge		Sn	5.95
As	5.8	Sb	77.5
Se	24.0	Te	38.9
Br	21.9	𝔍	46.5

With the exceptions of As and Sn the atomic susceptibility increases from the south toward the north of the graphic representation. While in the strongly magnetic metals the susceptibility decreases as we move on the diameter outward, the diamagnetic susceptibility increases when we travel in the same direction. Oxygen occupies an exceptional position through its paramagnetic properties. Its regular diamagnetic properties appear only in some of the organic and inorganic compounds.

No theory of magnetism is complete, which is unable to account for the exceptionally high magnetic constants of iron, nickel and cobalt and of the other few ferromagnetic substances like the Heusler alloys. All elements can be divided into an electropositive and an electronegative group; all elements are either para- or diamagnetic. Just as we can try to ascribe the forces of affinity to electrical charges in the atom, we might try to reduce affinity to magnetic forces, or magnetons. It is very interesting to note that the strongest positive metals of the alkali group are the weakest paramagnetic elements; and that the most negative elements like F, Cl, Br, 𝔍, are rather strongly diamagnetic. While the chemical properties of the most electropositive and electronegative elements may be explained by electrical forces, it seems possible to think that the magnetic forces due to magnetic doublets play a similar rôle in the chemical affinity of the elements with strongly magnetic properties. In this way we should get a periodicity of the magnetic properties as functions of the atomic weight as we have a periodic variation of the electropositive and negative properties of the elements. The graphical presentation of the law of periodicity shows the strongly magnetic metals just opposite to the strongly positive and negative metals. This explanation of the periodic variation of the magnetic properties would obtain strong support if it were possible to prove that all magnetons are identical just as all electrons are identical. But there is very little evidence in favor of the identical nature of all magnetons or elementary magnets as we shall see in the last paragraph. In three investigations[1] published in this journal the moments of the elementary magnets and the charge e have been determined for the following substances.

[1] The Absolute Values of the Moments of the Elementary Magnets of Iron, Nickel, and Magnetite, PHYSICAL REVIEW, Vol. XXX., p. 359, 1910. Stifler, PHYSICAL REVIEW, Vol. XXXIII., p. 268, 1911. P. Gumaer, PHYSICAL REVIEW, Vol. XXXV., p. 288, 1912.

	$m10^{20}$	$e \cdot 10^{20}$
Fe	5.15	1.60
Fe_3O_4	2.02	0.90
Ni	3.65	1.54
Co	6.21	1.56
Heusler alloy 1	3.55	1.54
Heusler alloy 2	4.23	2.04

$$1.53 \cdot 10^{-20} = e \text{ (average)}.$$

This value of e agrees fairly well with the values obtained by independent methods. On account of the necessary extrapolations it is difficult to obtain higher accuracy.

While I used the Langevin-Weiss theory for the determination of the elementary moment m, P. Weiss himself measured the intensities of magnetization at very low temperatures, and found noticeable deviations between the theory and the experiment in the temperature-intensity curve and he found at the same time a common divisor among the molecular intensities of the ferromagnetic substances. He called that divisor the magneton-gram, for which he gave the value 1,123.5. In addition the paramagnetic susceptibility of Fe_3O_4 above the critical temperature showed discontinuities as function of the temperature, which consisted of four straight lines, each of which led to a new determination of the magneton. Finally P. Weiss applied the equation

$$C_m = X_m T = \frac{\sigma_{m0}^2}{3r}$$

to solutions of paramagnetic substances containing iron and to a considerable number of solid salts. It has been shown, however, by Koenigsberger and Meslin that the molecular coefficient of magnetization of dissolved substances is a function of the concentration; at least for some solutions, while for others it seems to be constant. This fact makes it necessary to study solutions infinitely dilute or undissolved substances. The number of magnetons found by Weiss in the various substances is shown by the following series, in which the values coincide nearly with whole numbers.

10.41	28.83
21.89	26.99
21.96	28.94
24.04	29.19
28.03	21.23
27.93	25.05
30.09	17.97
25.99	20.04
27.11	12.12
27.91	20.16
27.69	20.16

If we displace the decimal point by one cypher to the left, we find again approximately whole numbers, which with one exception are almost as exact as the numbers given by P. Weiss. The numbers of magneton per molecule is rather high and it is not very surprising that in dividing for instance, 32,400 of $FeCl_3$ by 1,123.5 one finds approximately a whole number, 28.83 or 2.883 respectively. The large number of magnetons shown by the last two columns raises the question as to why those substances are so weakly magnetic, while nickel, being ferromagnetic, possesses at low temperatures only 3 magnetons. In addition, the numbers given by P. Weiss are based on the assumption that the magnetization of the pure salts and of the solutions varies according to Curie's law down to the absolute zero. As far as I know, this assumption has not yet been tested by experiments. Recently however Auguste Piccard[1] measured with great accuracy the susceptibility of oxygen at 20° C. and found for the moment of the atom: $7.8725 \cdot 10^1$; dividing this number by 1123.4 one gets 7.007, a whole number again. But in this case H. Kamerlingh Onnes and A. Perrier have shown that at low temperatures the susceptibility of oxygen changes according to the law:

$$X_{\text{lig}} = \frac{2284}{\sqrt{T}} 10^{-6},$$

$$X_{\text{sol}} = \frac{1690}{\sqrt{T}} 10^{-6}.$$

If this element does not follow the law of Curie, solid salts and solutions will probably also show deviations, and at the same time the evidence in favor of the magneton will decrease. At all events the number of magnetons seems to vary in the atom of a given element like nickel, which contains 3 magnetons at low temperature, 8 at high temperatures, 9 at the limit of the alloys of iron and nickel, 16 in the solutions. In order to determine the magneton P. Weiss, abandoning the theory, has directly measured the molecular moments of iron, nickel and cobalt; Auguste Piccard, on the contrary, has used the theory in order to find the magneton in spite of the measurements of Kamerlingh Onnes and A. Perrier. If we assume that Curie's law holds down to the absolute zero, we find for the elementary moment of oxygen

$$m = 2.58 \cdot 10^{-20}.$$

The moment of each individual magneton of Weiss on the other hand would be equal to:

$$\frac{1123.5}{6.1 \cdot 10^{23}} = 1.85 \cdot 10^{-21}.$$

[1] Archives de Genève, Tome XXXV., p. 480, 1913.

If we determine the molecular moments by means of Langevin's theory, we find values varying from $2.02 \cdot 10^{-22}$ to $5.15 \cdot 10^{-20}$ for substances so different among each other as oxygen, iron, magnetite and Heusler alloys. These values are of the order of magnitude which we should expect from the theory of quanta by Planck. Let us assume that the kinetic energy of a revolving electron $\frac{1}{2}mv^2$ is equal to a whole number z times hn, then we find:

$$\frac{m\omega^2 r^2}{2} = zhn = \frac{h\omega z}{2\pi},$$

$$\omega r^2 = \frac{hz}{\pi m}.$$

The moment of M of the revolving electron will be equal to:

$$M = iA = \pi r^2 \frac{e}{T} = \frac{e}{m}\frac{hz}{2\pi},$$

if we put $z = 1$, we find $M = 1.83 \cdot 10^{-20}$; for the frequency n we find $1.63 \cdot 10^{15}$, assuming $r = 1.5 \cdot 10^{-8}$. Why are these magnetons not sources of light? Not much importance must be attached to this approximate coincidence of $1.83 \cdot 10^{-20}$ with the moments determined by means of Langevin's theory. We find indeed about the same magnetic moment without the theory of the quanta, by calculating the velocity of the electron by the equation:

$$\frac{mv^2}{r} = \frac{e^2}{r^2};$$

putting $r = 1.5 \cdot 10^{-8}$; we get $n = 1.4 \cdot 10^{15}$; $M = 1.54 \cdot 10^{-20}$. And the question arises again, why does such a magneton not emit light? The difficulty might be removed by admitting a large number of electrons revolving in a circle, instead of one electron. The assumption of one single magneton, identically the same in all substances, seems to require much more experimental support, if it exists at all.

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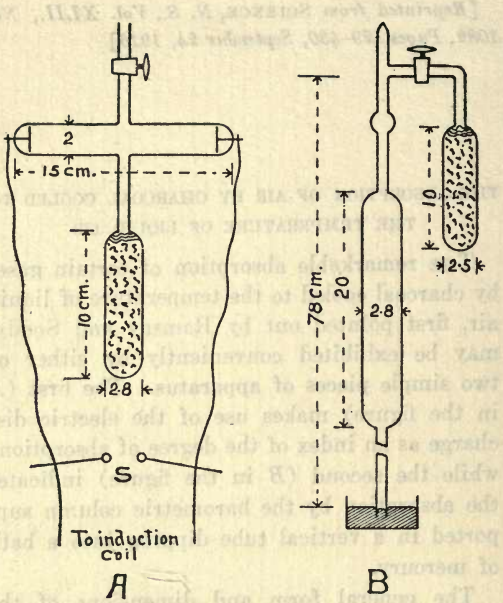
THE ABSORPTION OF AIR BY CHARCOAL COOLED TO THE TEMPERATURE OF LIQUID AIR

THE remarkable absorption of certain gases by charcoal cooled to the temperature of liquid air, first pointed out by Ramsay and Soddy, may be exhibited conveniently by either of two simple pieces of apparatus. The first (*A* in the figure) makes use of the electric discharge as an index of the degree of absorption; while the second (*B* in the figure) indicates the absorption by the barometric column supported in a vertical tube dipping into a bath of mercury.

The general form and dimensions of the discharge-tube and its attached charcoal bulb are indicated in *A*. The volume of the charcoal used should be approximately equal to that of the discharge tube proper. A vent closed by a valve is included. For the experiment to be in its best form the cocoanut charcoal should be freshly burned, and to prevent undue absorption of air when not in use the tube should be partially pumped out and the valve closed. The connections are made as shown in the figure, in which *S* is an alternative spark gap of about one centimeter length in parallel with the discharge tube. Any induction coil about the laboratory will answer. To operate, open the valve, then close it tightly, thus allowing the pressure within the tube to become atmospheric. On starting the induction coil the spark will pass at *S*. Now gently submerge the charcoal bulb in liquid air. In about one minute the spark at *S* will begin to weaken and a stringy discharge will appear between the electrodes of the discharge



tube. Soon the spark at *S* will cease while the tube will be filled with the characteristic Geissler tube glow. In about four minutes the walls of the discharge tube will begin to flouresce, due to the bombardment of cathode rays. The intensity of this fluorescence will rapidly increase and soon the entire tube will be uniformly filled with a beautiful apple-green color. In about one minute more, five minutes from the start, the greenish color will begin to fade and sparking will *reappear* at *S*, showing that the vacuum in the tube is becoming "hard." In short the pressure may thus be reduced from atmospheric to about .001 mm. mercury in five or six minutes with no other agency than that of the absorption of air by charcoal cooled to the temperature of liquid air.

The second method of showing the absorption of air, due to Dr. L. T. Jones, is at once clear by an inspection of *B* in the figure. The vertical stem, up to the branch leading to the charcoal bulb, should be at least 78 cm.

long. This stem may also have an enlargement about half way up as shown. A valve should be included to protect the charcoal when not in use. Before starting the experiment the valve is opened and the tube mounted in a bath of mercury. Liquid air is then applied to the charcoal bulb. The absorption proceeds slowly at first, but soon gains headway as the charcoal cools. The speed that the mercury column acquires as it rises up through and fills the enlargement is surprising. Even with the ratio of volume of tube to charcoal as shown in the figure (approximately 4:1) the mercury column will mount to nearly full atmospheric pressure in the short space of five or six minutes.

Added interest is to perform the two experiments simultaneously.

CHAS. T. KNIPP

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
August, 1915

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Carl T. Knipp

Laboratory of Physics,
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August, 1915.

Acoustics of Auditoriums.

INVESTIGATION OF THE ACOUSTICAL PROPERTIES OF THE ARMORY AT THE UNIVERSITY OF ILLINOIS.

By F. R. WATSON, *Associate Professor of Physics, University of Illinois.*

(Reprinted from The Brickbvilder, October, 1915.)

THE Armory at the University of Illinois presents an unusual case of defective acoustics because of its very large volume and comparatively small absorbing power. It was built to fulfil the usual requirements of an armory in regard to military drills; but, in addition, it has been used on several occasions for convocations and assemblies where the audiences have been very large. The acoustics proved to be impossible for speaking and music. In view of the proposed continued use of the building for such assemblies, the writer carried on an investigation to determine the possibilities of making it satisfactory in its acoustical properties.

The Armory is 400 feet long, 212 feet wide, and 93 feet to the highest point of the roof. Acoustically, it is defective because of echoes and reverberation. Echoes are

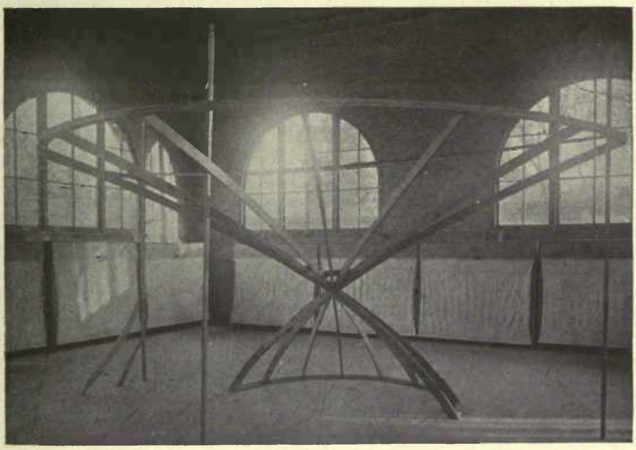

Fig. 1. Framework of Parabolic Reflector

set up by the distant walls, while the reverberation is caused by the undue prolongation of sound.

Several experiments were tried to determine the value of special devices for reinforcing and directing the sound. In one case, a huge parabolic reflector of special construction was used. This was based upon the known action of parabolic reflectors in directing sound along the axis of the parabola.*

A modified paraboloid was constructed, the parabolic ribs of which were arranged so as to spread the reflected sound over the entire area occupied by the audience. The framework, pictured in Fig. 1, was covered with oilcloth and mounted over the head of the speaker so that his mouth was at the common focus of all the parabolic ribs. Preliminary tests with the reflector showed that it admirably fulfilled its purpose in

* "The Use of Sounding Boards in an Auditorium," *Physical Review*, Vol. 1 (2), p. 241, 1913, and THE BRICKBVILDER, June, 1913, and August, 1913.

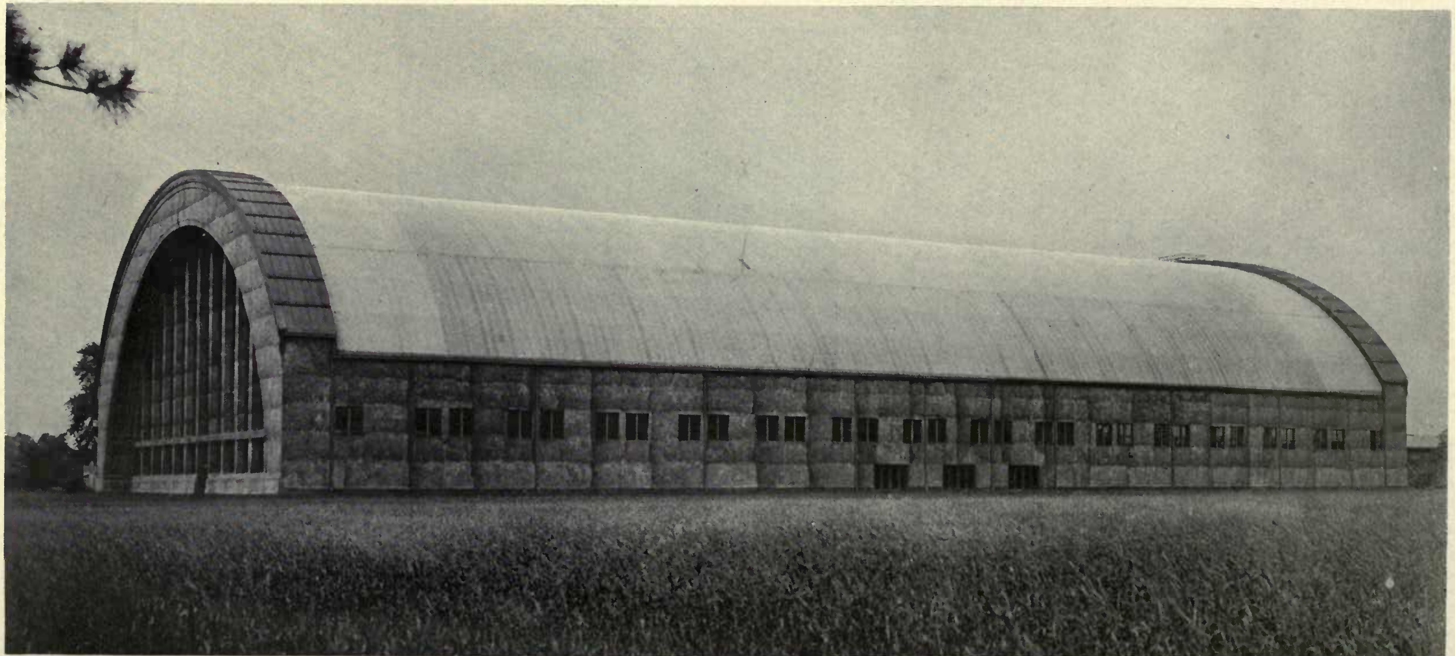

Fig. 2. The Armory at the University of Illinois

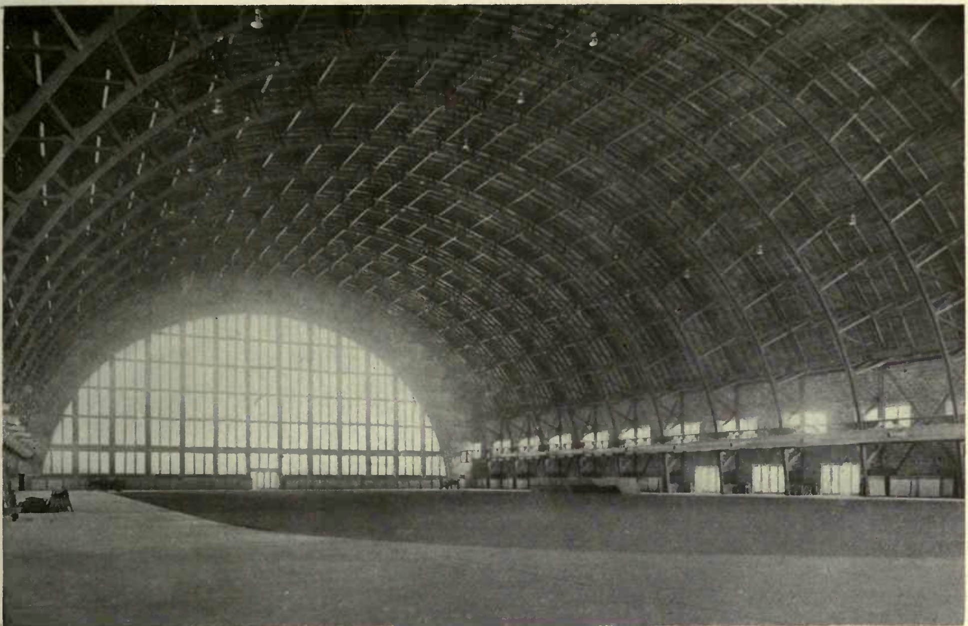

Fig. 3. Interior of Armory

Fig. 4. Interior Arranged for Commencement Exercises

directing sound; but when used at an assembly with an audience, its action was tically drowned out by the excessive reverberation which prohibited any possibil satisfactory acoustics.

Another experiment of like nature involved the use of a special megaphone to di ute the sound of the speaker's voice. This megaphone was more efficient tha reflector, since it utilized all the sound sent out by the speaker instead of onl portion intercepted by the reflector. This device was also of little benefit beca the excessive reverberation.

A third trial was made by using a number of loud-speaking telephones at dif positions in the Armory. This attempt was also unsuccessful, although the telep when used out in the open air were very effective in reinforcing and directing the s

These experiments showed the impossibility of using the entire Armory for spe purposes unless the reverberation could be materially reduced. The investigatio then directed to the determination of the constants of reverberation and the poss of correcting them. Sabine's method* was used for this purpose. His formu reverberation is expressed as follows:

$$t = kv \div a,$$

where t is the time of reverberation, v the volume of the room, a the sound-abso power of all the exposed surfaces in the room, and k a constant which is deter experimentally. Applying this formula to the case of the Armory, the volume of is 6,652,000 cubic feet, and the total absorbing power, without an audience, units, the time of reverberation was calculated to be 24 seconds. This value is u ally large. The Auditorium at the University of Illinois, seating 2,200 people, reverberation before its acoustical correction of 9 seconds and was considered very bad.† The conditions in the Armory by comparison with this case may be in to be exceptionally unsatisfactory.

Calculations made to ascertain the effect of introducing sound-absorbing ma showed that the installation of 50,000 square feet of hairfelt would reduce the reve tion to 4.66 seconds, a value which would still be too large for satisfactory spea The only alternative was to reduce the volume. Calculations were then made f acoustical properties of a room partitioned off by canvas curtains at one end Armory so as to enclose a space 212 feet by 134 feet and 35 feet high. To do was first necessary to determine experimentally the action of the canvas in transm and absorbing sound. The time of reverberation for the room with an audie 4,500 people present was then estimated to be 1.1 seconds, a value which has found by repeated experience to be satisfactory.

On the basis of this calculation a room of the specified dimensions was enclo one end of the Armory and used for the University Commencement exercises. Fig. 4.) Auditors in all parts of this canvas-enclosed room could hear and unde the various speakers, so that the room was considered a success from the standpo acoustics.

A further step to be undertaken in the investigation lies in the proposed instal of some sound-absorbing materials upon the walls of the Armory itself. It is that by this means the time of reverberation may be reduced to a reasonable leng make the building entirely satisfactory for military drills and band concerts. W or not it will also be suitable for assemblies where there is speaking, remains to be

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* *American Architect*, 1900.

[Reprinted from the PHYSICAL REIVEW, N.S., Vol. VI. No. 5, November, 1915.]

SATURATION VALUE OF THE INTENSITY OF MAGNETIZATION AND THE THEORY OF THE HYSTERESIS LOOP.

BY E. H. WILLIAMS.

RECENTLY, Weiss[1] has shown that an alloy composed of iron and cobalt combined in relative amounts given by the expression Fe_2Co gives a much higher value of the intensity of magnetization than either iron or cobalt taken alone. Furthermore, it has been shown by Mr. D. T. Yensen[2] that the magnetic properties of pure iron can be greatly improved by melting the iron in a vacuum and it was hoped that the magnetic properties of Fe_2Co could be improved by treating it in like manner. The object of the present paper was not only to make a careful study of the saturation value of the intensity of magnetization of Fe_2Co prepared under various conditions but to use the data thus obtained in a test of the theory of the hysteresis loop as developed by J. Kunz.[3]

In his paper, Kunz tests the theory with the data then available. The results show the discrepancy between theory and experiment to be very great. If all quantities involved are obtained from the same sample, the test of the theory will be more satisfactory.

PREPARATION OF SAMPLES.

Iron and cobalt were taken in the proportion indicated by the formula Fe_2Co, melted in a vacuum furnace under pressures varying from 5 mm. to 0.5 mm. Hg, allowed to cool slowly and then forged into long bars from which samples were turned. In most cases enough of the material was taken to make both an ellipsoid and a rod—the ellipsoid for the determination of the saturation value of the intensity of magnetization and the rod for the determination of the hysteresis loop. The hysteresis data were taken by Mr. D. T. Yensen on a magnetic testing apparatus similar to those used by the Bureau of Standards. Mr. Yensen has made a study of the samples from the viewpoint of the engineer. His results are to be published soon in the General Electric Review.

[1] P. Weiss, Compt. Rend., 156, p. 1970, 1913.

[2] D. T. Yensen, Bul. No. 72, Eng. Exp. Sta., Univ. of Illinois, Urbana, Ill.

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directing sound; but when used at an assembly with an audience, its action was practically drowned out by the excessive reverberation which prohibited any possibility of satisfactory acoustics.

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† "Acoustics of Auditoriums," Bulletin No. 73 of the University of Illinois, Engineering Experiment Station.

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Ellipsoids.

A great deal of trouble was experienced in making ellipsoids that were accurate. The form of the ellipsoids was tested by projecting an image of the ellipsoid on the figure of an ellipsoid drawn to the desired proportions. Finally the accuracy was tested by comparing the volumes obtained by calculation, using the dimensions of the ellipsoid, with those obtained by immersion in distilled water at known temperature. No ellipsoid was used where the difference in volume differed by more than 2 per cent. and most of them differed by less than 1 per cent. The ellipsoids were about 1.19 cm. in length and about .56 cm. in diameter.

The author wishes to express his thanks to P. Weiss for samples of his material which he kindly sent. This material, when received, was porous and had apparently been melted and cast at atmospheric pressure. One ellipsoid was turned from the material just as received. A second ellipsoid was turned from a portion of the material which had been forged into a small rod, after which the remainder of the sample was remelted in a vacuum furnace under a pressure of .5 mm. of Hg. It was then forged into a rod from which an ellipsoid was turned. The results obtained with these ellipsoids are included in Table I.

The field inside an ellipsoid is uniform and is given by

$$H = H_0 - NI, \tag{1}$$

where H_0 is the external field applied, I the intensity of magnetization, H the resultant field within the ellipsoid and N a constant depending on the dimensions of the ellipsoid.

The field H_0 was produced by a large electromagnet the pole pieces of which were 3.2 cm. apart and bored to receive a glass tube 9 mm. in diameter. On this tube was wound an induction helix. The field, H_0, between the poles of the magnet was calibrated by two methods—by means of a flip coil and with a magnetic balance. The mean of the two calibrations, which differed in no case by more than one half of one per cent., was taken to plot the calibration curve. The intensity of magnetization I was obtained by suddenly removing the ellipsoid from the induction helix between the pole pieces and noting the change of flux as indicated by a ballistic galvanometer. From the constants of the apparatus the value of I could be calculated.

If, in equation (1), NI is greater than H_0, H becomes negative while H_0 and I are still positive, *i. e.*, the field within the ellipsoid is opposite in direction to the field outside. The ellipsoids used in this work were such as to produce this result, so that when a hysteresis loop was taken with one of the ellipsoids a very peculiar S-shaped form was obtained.

Results for I_m.

The results for the saturation values of the intensity of magnetization are summarized in Table I. In this table also, values obtained by other experimenters as well as by the author are given for comparison. Annealing these samples at 900° C. and 1100° C. produced practically no change in the values of I_m. Analysis of the first two samples of Fe_2Co listed in Table I., were made, the first showing 33.36 per cent. Co and the second 33.33 per cent. Co. From these results we see that com-

Table I.

Values of I_m (t = 20° C.)

Commercial steel (Williams)	1,751
Swedish wrought iron (Ewing)	1,690
Bessemer steel (.4 per cent. C.)	1,770
Electrolytic iron (melted under pressure of 3 mm. of Hg (Williams)	1,798
Cobalt (1.66 per cent. Fe) (Ewing)	1,310
Cobalt (pure) (Stifler)	1,421
Cobalt (melted under pressure of 1 mm. Hg (about 99 per cent. pure) (Williams)	1,504
Fe_2Co melted under pressure of 3 mm. of Hg without being forged (Williams)	1,791
Same—hand forged (Williams)	1,962
Same—forged with steam hammer (Williams)	1,977
Fe_2Co melted under pressure of 1 mm. of Hg. Forged with steam hammer (Williams)	2,050
Fe_2Co melted under pressure of 0.5 mm. of Hg. Forged with steam hammer (Williams)	2,056
Fe_2Co melted and cast at atmospheric pressure (sample received from P. Weiss) (Williams)	1,752
Same—forged as received (Williams)	1,977
Same—remelted under .5 mm. pressure and forged (Williams)	2,038

bining pure iron for which the value of I_m is 1,800, when the iron is melted in a vacuum, with cobalt for which the value of I_m is 1,500 when melted under the same conditions, we obtain an alloy for which I_m is 2,050, or 14 per cent. higher than pure iron itself. Weiss, in the paper referred to above, states that if one takes into account the difference in atomic weight, the temperature at which ferromagnetism disappears and the densities, one finds that at ordinary temperatures ferro-cobalt has a magnetization at saturation 10 per cent. higher than that of iron, so that the extra 4 per cent. is probably due to the fact that the alloy in the present case was melted in a vacuum. This conclusion is sub-

stantiated by the difference between the last two results of Table I. (material received from P. Weiss). This difference is undoubtedly due to melting under greatly reduced pressure since all other conditions are as nearly equal as it was possible to make them.

Photomicrographs of the first two samples of Fe_2Co given in Table I. are shown in Figs. 1, 2, 3 and 4. Fig. 1 is of the first sample after being forged and Fig. 2 is of the same sample after being annealed at 900° C. and cooled uniformly at the rate of 30° C. per hour. Fig. 3 is of the second sample of Fe_2Co listed in Table I. after the same had been forged and Fig. 4 is the same sample after being annealed at 900° C.

Hysteresis Theory.

In the article by J. Kunz referred to above, the author obtains the following expression for the energy of the hysteresis loop:

$$W = 2I_mH_c^2\left[\frac{1}{H_c\sqrt{\dfrac{I_m}{I_m + I_1 + \Delta I_1}}} - \frac{1}{H_1 - \Delta H}\right] + I_mH_c^2\Delta H\left[\frac{1}{H_c^2\dfrac{I_m}{I_m + I_1 + \Delta I_1}} - \frac{1}{(H_1 - \Delta H)^2}\right],$$

where I_m is the saturation value of the intensity of magnetization, H_c the coercive force, I_1 the intensity of magnetization corresponding to the magnetizing field H_1, and where

$$H = H_c\left(1 - \frac{1}{\sqrt{2}}\right)$$

and

$$I_1 = I_m\left[\left(\frac{H_c}{H_1 - \Delta H}\right)^2 - \left(\frac{H_c}{H_1}\right)^2\right].$$

According to this theory the hysteresis loss per cycle experienced when the field alternates between the values $+ H_1$ and $- H_1$, producing the intensities of magnetization $+ I_1$ and $- I_1$, can be calculated directly if one knows the values of I_m and H_c for the material concerned.

As pointed out above, the test given this theory proved very unsatisfactory and seemed to indicate that the theory was of very little practical importance.

It seemed desirable to give the theory a thorough test by the careful determination of the four quantities I_m, H_c, H_1 and I_1 with the same sample. The results for the hysteresis loss, W, calculated and the hysteresis loss, W', as measured from the hysteresis loops are given in

PHYSICAL REVIEW, VOL. VI., SECOND SERIES.
November, 1915.

PLATE I.
To face page 408.

Fig. 1.

Fe_2Co melted under 3 mm. pressure and forged.

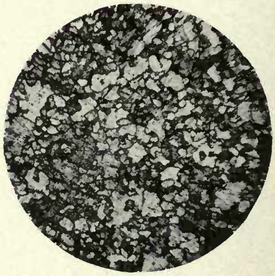


Fig. 2.

Same as Fig. 1, annealed at 900° C.

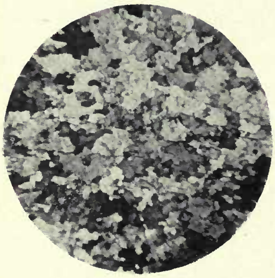


Fig. 3.

Fe_2Co melted under 1 mm. pressure and forged.

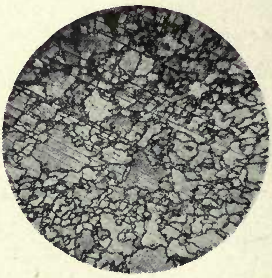


Fig. 4.

Same as Fig. 3, annealed at 900° C.

E. H. WILLIAMS.

Tables II., III. and IV. Table II. is for a sample of Fe_2Co before being annealed; Table III. for the same sample after being annealed at 900° C.,

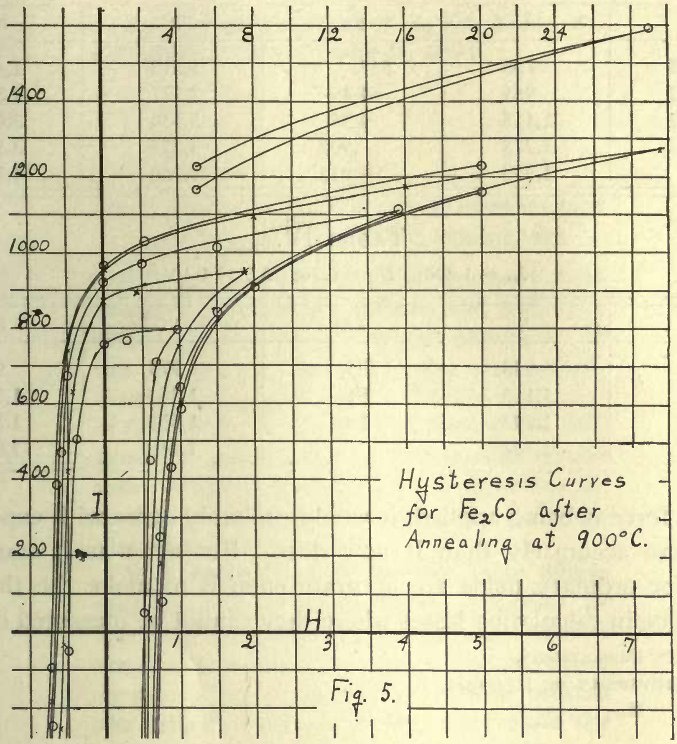


Fig. 5.

and Table IV. for a sample of pure iron after being annealed at 900° C. The hysteresis curves from which the values of W' in Table III. were measured are shown in Fig. 5.

TABLE II.

$I_m = 2{,}050$; $H_c = 6.4$; $\Delta H = 1.876$.

H_1	I_1	ΔI_1	W	W'
14	415	143.5	19,580	18,685
24.5	798	24.2	28,700	28,600
56.5	1,179	2.27	35,620	41,825
146.	1,588	.06	45,630	56,245

The results in Tables II., III. and IV. show fairly good agreement between the values for the hysteresis loss as calculated by the above formula and those obtained by measurement of the hysteresis curves.

If the theory were modified to take into account the curvature as the

TABLE III.

$I_m = 2{,}050$; $H_c = 0.65$; $\Delta H = 0.19$.

H_1	I_1	ΔI_1	W	W'
.95	796	537.	1,513	1,488
1.82	955	64.4	2,727	2,340
3.82	1,115	6.35	3,434	3,020
7.33	1,273	.902	3,777	3,792
29.	1,590	.013	4,166	4,930

TABLE IV.

$I_m = 1{,}800$; $H_c = 0.36$; $\Delta H = 0.105$.

H_1	I_1	ΔI_1	W	W'
.6	955	306.	973	970
0.9	1,115	81.	1,364	1,353
1.5	1,195	16.	1,651	1,552
6.5	1,264	.2	1,940	1,918

coercive force is being applied, it would probably agree with experiment even more accurately than it now does. But even as it stands the results for ordinary fields are accurate enough to make the theory of great value in calculating losses where they cannot be measured directly.

PHYSICS DEPARTMENT,
UNIVERSITY OF ILLINOIS,
May, 1915.

[*Reprinted from* SCIENCE, *N. S., Vol. XLII., No. 1096, Pages 942–943, December 31, 1915*]

COLOR EFFECTS OF POSITIVE AND OF CATHODE RAYS IN RESIDUAL AIR, HYDROGEN, HELIUM, ETC.

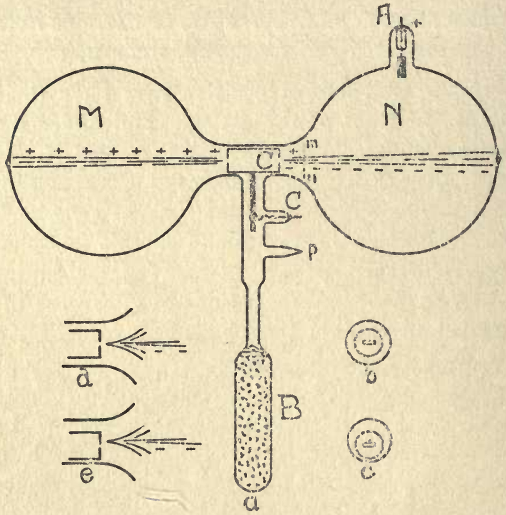
As is well known positive rays have their origin in front of the cathode, and under the action of the electric force fall toward it. If the cathode is perforated the rays stream through and constitute the "kanal strahlen" of Goldstein. Tubes built to exhibit this phenomenon form a part of the regular equipment of nearly all collections of apparatus intended to exhibit the phenomena of electric discharge through gases.

Most beautiful and striking color effects may be had by using hollow cathodes[1] in specially designed tubes containing each a trace of some inert gas such as helium, argon or neon. The color effect is striking because the cathode beam is of one color, while the positive ray beam in the same gas is of an entirely different color. The general design of the tubes that Dr. Jakob Kunz and the writer have found best suited is shown in the accompanying figure. The discharge tube is dumb-bell shaped. It is made of two 2 liter Florence flasks, *M* and *N*. The hollow cylindrical cathode *C* is mounted in the neck, while the anode *A* is placed in one of the bulbs. The cathode terminal *C*, the nipple *p* for exhausting, and the charcoal bulb *B* are all attached to one vertical tube as shown.

The process of filling the discharge tube, sealing it off from the pump, and its subsequent use is as follows: After the tube is con-

[1] J. J. Thomson, "Rays of Positive Electricity," p. 6, 1913.

structed, and the charcoal bulb B attached, the exhaust nipple is put in communication with a pump, and also to some source of the gas to be used. During the early part of the exhaustion it is well to gently heat the bulb B. Continue the pumping until the tube on sparking shows a tendency of becoming hard.



As this stage is approached cathode rays will appear as a compact beam in the bulb N, while a beam of positive rays will traverse the bulb M. Now admit a small quantity of the desired gas, say, helium. The chances are that too much gas will enter the discharge tube and thus destroy the definition of the two beams. To restore it pumping should be continued and at the same time the bulb B should be carefully submerged in liquid air. Care must be exercised not to reduce the content of helium by too long continued pumping. The cooled charcoal will absorb the traces of air leaving the tube MN relatively richer and richer in helium—since helium, an inert gas, is but slightly absorbed by the cooled charcoal. The cathode beam in N as well as the positive ray beam in M will each increase in bright-

ness and definition, reaching a maximum, after which, as the process continues, they will begin to fade. At the stage when the beams are judged brightest the exhaust nipple *p* is sealed off from the pump. The tube is now in its finished state. Removing the liquid air, the charcoal gives up its absorbed gas and the beams weaken and become diffused. For subsequent use it is only necessary to submerge *B* in liquid air while the discharge from an induction coil is passing. The beams in *M* and *N* will increase in brightness and definition as the absorption of the active gases proceeds, thus giving ample time for the observation of the changes going on within the tube.

The most interesting phenomenon is the *color* of the two beams. The cathode beam in helium is a greenish gray color, while the positive ray beam in the *same* gas is a beautiful red. There is no mistaking the colors. Indeed the red due to the positive ions is so persistent that it appears at the very origin of these rays—at the edge of the Crookes dark space in front of the cathode (shown by the dotted line *mn* in the figure).

The usefulness of the above described tube for many laboratories is limited because liquid air is used in its initial adjustment and subsequent operation. If desired the bulb *B* may also be sealed off. The only disadvantage is that this fixes the gas content in the tube. In case no liquid air is available it is still possible to construct the tube provided access may be had to a good pump. In this event the discharge tube should be washed out several times with the desired gas, in order to remove every trace of air, and then sealed off when the beams are brightest. This gives a permanent tube provided the occluded gases in the electrodes and walls of the vessel do not in time let the vacuum down. Danger from this source, however, may be largely avoided by gently heating the tube during exhaustion.

The obvious advantage of a charcoal bulb is that the proper exhaustion can always be reached and at the same time the discharge at various stages of exhaustion successively exhibited.

It should be added that the best results only are obtained when the hollow cathode *C*, which is an aluminum cylinder closed at the ends with aluminum discs through the center of each is cut a rectangular opening about 1 mm. by 6 mm., is placed exactly on the axis of the tube connecting the bulbs *M* and *N*. The correct position is shown in the figure, end view at *b*, and side view at *d*. The discharge leaving the cathode, confined in a narrow tube as here, is always along the axis of the glass tube, regardless of the alignment of the cathode. In other words, the shape of the glass tube rather than the shape of the cathode determines the position of the cathode beam. Lack of alignment is shown at *c* and *e* where the opening through the hollow cathode is below the axis and as a result few positive rays get through and show in the bulb *M*, though they show distinctly at their origin in front of the cathode. To avoid possible lack of alignment it is advised to make the hollow cathode *C* of such diameter so as to fit snugly into the neck connecting *M* and *N* as shown in *a* of the figure.

An interesting test to show that the beam in *N* is composed of electrons, and that in *M* of positively charged ions, is to deflect them in turn by a strong electro-magnet. The cathode beam is readily deflected while the positive ray beam is but little deflected and that in the opposite sense. This is in full agreement with the theory of the magnetic deflection of moving positive and negative charges.

CHAS. T. KNIPP

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October 9, 1915

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THE STRUCTURE OF γ RAYS ON THE BASIS OF THE ELECTRO-MAGNETIC THEORY OF LIGHT.

BY JAKOB KUNZ.

THE difficulties in the theories of radiation have become so great in the last few years that only a very fundamental new idea will be able to bring harmony into the present chaos of facts and theories. Before that solving idea appears, we can only draw all the logical conclusions which follow from a given hypothesis and compare these conclusions with experiments. This procedure leads to the result that neither one of the present hypotheses is able to coördinate satisfactorily all the facts. The electromagnetic undulatory theory of light explains the phenomena of reflection, refraction, interference, diffraction, etc., but it fails to explain the phenomena of radiation, the photoelectric and the related effects. The corpuscular theories and the quantum theories on the other hand are especially invented for the explanation of the latter group of phenomena, failing to account for the first group. Not only are the fundamental assumptions of the two theories different, but the older electromagnetic undulatory theory gives not only mathematical relations between the different quantities involved, but it visualizes at the same time, the phenomena, which are explained, so that we can see the mechanism of the processes. The recent quantum theories on the other hand give us only algebraic relations between different quantities such as the law of the photoelectric effect $\frac{1}{2}mv^2 = hn - V_0$, without giving us the least idea as to the mechanism of the phenomenon. It seems to be advantageous if not necessary for a theory to visualize the phenomena. Led by this idea, I shall show by the following figures and calculations that the electromagnetic theory of very hard Roentgen and γ rays leads to conclusions which remind us of an emission or corpuscular theory of rays. The problem has already been solved by A. Sommerfeld,[1] "Uber

[1] Sitzungsberichte der K. B. Akademie der Wissenschaften, 41, p. 1, 1911.

die Structur der γ Strahlen." I shall use, however, a different method which allows the visualizing of the remarkable conclusion of the electromagnetic theory.

An electron moves with the velocity v in the x direction; the electric force E at a distance r from the charge making an angle φ with the direction of motion is equal to

$$E = \frac{e\left(1 - \frac{v^2}{c^2}\right)\sqrt{x^2 + y^2 + z^2}}{\left[x^2 + (y^2 + z^2)\left(1 - \frac{v^2}{c^2}\right)\right]^{\frac{3}{2}}}.$$

x, y, z are the coördinates of the point in which the electric force E is measured, the zero of the system of the coördinates lies in the electron. We can express E also in the following way

$$E = \frac{e\left(1 - \frac{v^2}{c^2}\right)}{r^2\left(1 - \left(\frac{v^2}{c^2}\right)\sin^2\varphi\right)^{\frac{3}{2}}}$$

The magnetic force H is given by

$$H = \frac{Ev\sin\varphi}{c}.$$

These equations tell us that with increasing velocity v the electromagnetic field following the electron, is more and more compressed as it were toward the equatorial plane, being perpendicular to the direction of motion.

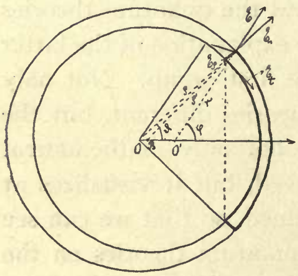

Fig. 1.

If now the electron comes into collision with a metallic plate it will come to rest during an interval of time t and move meanwhile over the distance l. If we draw the line of force t seconds after the collision began, we find the well-known disturbance within the Roentgen pulse, whose thickness δ is a function of ϑ given by the equation

$$\delta = \frac{l}{v}(2c - v\cos\vartheta).$$

Within the sphere with radius ρ' drawn round about B of Fig. 1 the final position of rest of the electron, we find the ordinary electrostatic field. Without the sphere with radius ρ, drawn about O, where the collision at

the moment $t = 0$ began, we find the electromagnetic field accompanying the electron in motion. The electric force E, pointing toward $0'$ where the electron at the moment t would be, if it had not been stopped, is equal to

$$E = \frac{e(1-\beta^2)}{r^2(1-\beta^2 \sin^2 \varphi)^{\frac{3}{2}}}; \quad \beta = \frac{v}{c}.$$

The component En of E in the direction ρ is equal to

$$E_n = \frac{e(1-\beta^2) r \cos(\varphi - \vartheta)}{r^3(1-\beta^2 \sin^2 \varphi)^{\frac{3}{2}}},$$

$$r \cos(\varphi - \vartheta) = \rho(1 - \beta \cos \vartheta),$$

$$r^3(1-\beta^2 \sin^2 \varphi)^{\frac{3}{2}} = \rho^3(1-\beta \cos \vartheta)^3,$$

$$E_n = \frac{e(1-\beta^2)}{\rho^2(1-\beta \cos \vartheta)^2}.$$

We shall now calculate the tangential electric force E_t in the shell. In Fig. 1 we consider the volume cut out of the shell by a cone with an angle ϑ. As there are no charges within this cap, the resultant flux of the electric force flowing out of the volume must be equal to zero. The inflowing flux ϕ_1 is due to the component E_t' and to the force e/ρ'^2 or approximately e/ρ^2

$$\phi_1 = E_t \delta 2\pi\rho \sin \vartheta + \frac{e}{\rho^2} 2\pi\rho(\rho - \rho \cos \vartheta),$$

$$= E_t \delta 2\pi\rho \sin \vartheta + e 2\pi(1 - \cos \vartheta).$$

The outflowing flux ϕ_2 is due to the force E_n and is equal to

$$\phi_2 = \int_0^{\vartheta} E_n 2\pi\rho \sin \vartheta d\vartheta \cdot \rho$$

$$= e(1-\beta^2) 2\pi \int_0^{\vartheta} \frac{\sin \vartheta d\vartheta}{(1-\beta \cos \vartheta)^2}$$

$$= e(1+\beta) 2\pi \frac{1 - \cos \vartheta}{1 - \beta \cos \vartheta}$$

$$\phi_1 = \phi_2,$$

hence

$$E_t \delta 2\pi\rho \sin \vartheta + e 2\pi(1-\cos \vartheta) = e(1+\beta) 2\pi \frac{1-\cos \vartheta}{1-\beta \cos \vartheta}$$

$$E_t = \frac{e\beta \sin \vartheta}{\delta\rho(1-\beta \cos \vartheta)} = \frac{ev \sin \vartheta}{\delta\rho(c - v \cos \vartheta)}.$$

In every electromagnetic disturbance the electrostatic energy is equal to the magnetic energy. This is the case if we assume for the magnetic

force H in the pulse the expression

$$H = cE_t.$$

The distribution of the lines of force is shown in Fig. 2, and in Fig. 3 for the limiting case where the velocity v becomes equal to the velocity

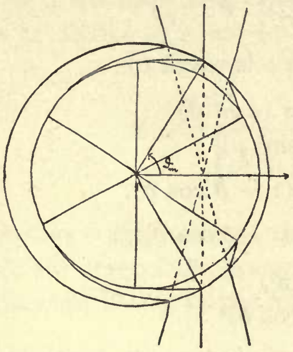

Fig. 2.

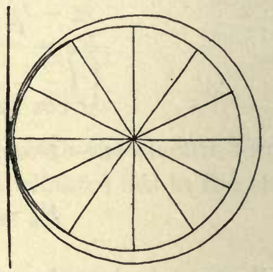

Fig. 3.

c of light. We see from these figures that with increasing velocity v the electric force E_t becomes a maximum in a certain direction ϑ_m where most of the energy flows away from the electron, and that in the limit for $v = c$ the energy flows out in the direction $\vartheta_m = 0$, that is in the direction of motion of the electron. When the velocity v is so small that it can be neglected with respect to c, then on the contrary, the maximum of the energy radiates away from the electron in the equatorial plane.

$$E_t = \frac{ev^2 \sin \vartheta}{l\rho(c - v \cos \vartheta)(2c - v \cos \vartheta)}.$$

If we neglect at first the change in the factor $1/(2c - v \cos \vartheta)$, we get

$$E_t = \frac{ev^2 \sin \vartheta}{l\rho(c - v \cos \vartheta)}$$

and

$$\frac{\partial E_t}{\partial \vartheta} = \frac{e}{l\rho} \frac{v^2(c \text{ vos } \vartheta - v)}{(c - v \cos \vartheta)^2},$$

which vanishes when $\cos \vartheta = v/c$; for this angle the electric force is a maximum and the energy radiating away from the electron becomes approximately a maximum also.

When $\frac{v}{c} = 0.9$ then $\vartheta = 25° 45'$,

when $\frac{v}{c} = 0.99$ then $\vartheta = 8°$ and when

$\frac{v}{c} = 1$ then $\vartheta = 0°$.

The effect becomes marked only when the velocity v approaches that of light. Returning to the complete expression E_t, we find that it becomes a maximum for the angle ϑ determined by

$$\cos^3 \vartheta - 2\left(1 + \frac{c^2}{v^2}\right)\cos \vartheta + 3\frac{c}{v} = 0.$$

For

$$\frac{c}{v} = \frac{10}{9},$$

the three roots are -2.42, 1.497 and 0.9210, hence the angle $\vartheta = 22° 55'$. But as the shell has not everywhere the same thickness, this angle does not accurately indicate the direction in which most of the energy is radiated away.

The energy of the electric force E_t within the elementary volume $2\pi\rho \sin \vartheta \rho d\vartheta\delta$ is equal to

$$dE_{es} = \frac{1}{8\pi} E_t^2 2\pi\rho \sin \vartheta\rho d\vartheta\delta$$

$$= \frac{1}{4} e^2 \frac{v^3 \sin^3 \vartheta d\vartheta}{l(2c - v\cos\vartheta)(c - v\cos\vartheta)^2}.$$

The electrostatic energy in the cap under the angle ϑ is given by the integral

$$E_{es} = -\frac{1}{4}\frac{e^2v^3}{l}\int_0^{\vartheta} \frac{\sin^2 \vartheta d\cos\vartheta}{(2c - v\cos\vartheta)(c - v\cos\vartheta)^2}$$

$$= \frac{1}{4}\frac{e^2}{l}\left\{\frac{3c^2 - v^2}{c^2}\log\frac{c - v\cos\vartheta}{c - v} - \frac{4c^2 - v^2}{c^2}\log\frac{2c - v\cos\vartheta}{2c - v}\right.$$

$$\left. + \frac{c^2 - v^2}{c}\left[\frac{1}{c - v\cos\vartheta} - \frac{1}{c - v}\right]\right\}.$$

The function

$$f(\vartheta) = \frac{\sin^3\vartheta}{(2c - v\cos\vartheta)(c - v\cos\vartheta)^2}$$

or

$$f(\vartheta) = \frac{\sin^3\vartheta}{\left(2 - \frac{v}{c}\cos\vartheta\right)\left(1 - \frac{v}{c}\cos\vartheta\right)^2}$$

has been plotted in Fig. 4 for the values $v/c = 9/10$ and $v/c = 99/100$. This figure shows clearly that with increasing velocity the energy is radiated away in a direction which approaches more and more the direction of motion of the electron. The bearing of this conclusion on the fluctuations of Schweidler and perhaps on the difference in the

photoelectric effect according to the incidence of the beam of light, or Roentgen rays is obvious. If we attribute to the electromagnetic field not only energy, but also momentum and mass, then it follows, that the electromagnetic mass of the electron, when it comes to rest, is thrown

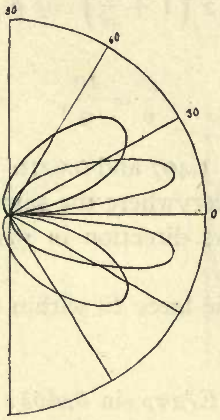

Fig. 4.

forward more and more with increasing velocity. This electromagnetic mass and momentum concentrated in a comparatively small space, is not so very different from the notion of light particles in the old emission theory.

UNIVERSITY OF ILLINOIS,
LABORATORY OF PHYSICS,
May 22, 1915.

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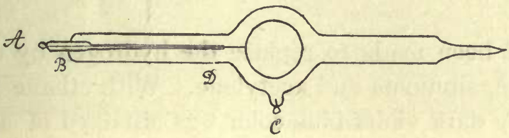
ON THE CONSTRUCTION OF SENSITIVE PHOTOELECTRIC CELLS.

BY JAKOB KUNZ AND JOEL STEBBINS.

THE high sensitiveness of photoelectric cells of alkalihydrides has been discovered by Elster and Geitel. For several years we have tried to apply this cell in stellar photometry. J. G. Kemp[1] and W. F. Schulz[2] have shown that it is possible and advantageous to replace the selenium in stellar photometry by the photoelectric cell. Practically at the same time corresponding measurements have been made in Germany, especially in the observatory of Berlin.

One of us has reported on an astronomical discovery made by the photoelectric cell in the Evanston meeting of the American Astronomical Society in September, 1914. In the last two years we have tried to improve the photometric properties of the cell and we have arrived at a form which seems to be satisfactory with respect to sensitiveness, constancy, absence of the dark current, etc. The final form is indicated by Fig. 1, which is drawn full size. The glass bulb is 3.4 cm. in diameter.



Fg. 1.

It contains a small platinum cathode C, a platinum ring of 1.8 cm. in diameter as an anode A, which passes through a platinum cylinder B; this cylinder was found to be very necessary in order to lead surface and electrolytic currents of the glass to earth. Strips of tinfoil were occasionally wrapped around the glass cylinder at D and the cathode C, in order that dark currents might be suppressed. The tubes are connected to the mercury pump and heated two to three hours to 330° C. to drive off the remaining gases. A small quantity of the pure alkali metal is distilled on the silver mirror of the cell, which is kept cool by cold water or ice, at the same time the end AD of the cell is heated from 160 to 240°, according to the alkali metal, by means of a heating coil.

[1] J. G. Kemp, PHYS. REV., Vol. I., p. 274, 1913.

[2] W. F. Schulz, Astrophysical Journal, Vol. XXXVIII, p. 187.

The most sensitive cells have been obtained when the metal was deposited in a thin uniform layer. Pure hydrogen from palladium was then admitted and its pressure so adjusted that a potential difference of 280 to 400 volts between the electrodes produced a uniform glow discharge. Often a spark or arc appears instead of the bright uniform glow, and the spark is apt to destroy the sensitive layer. By experience one finds the best conditions for the glow to appear. The sensitiveness will be tested during the formation of the hydride. As a rule the formation requires only one to three seconds for the maximum sensitiveness; if continued, the colors of the compound change and the deflection in the galvanometer decreases. During the formation the electrode C is negative and A positive. But in certain gases like ammonia and ethane a sensitive layer is also formed if the current is reversed. After the formation the gas is carefully pumped out and replaced by an inert gas, helium, argon, or neon. The pressure is so chosen as to get a maximum sensitiveness.

Experiments have been made with the object of finding out the influence of the size and shape of the cell on the sensitiveness. The diameter varied from 5 to 2.5 cm. and the sensitiveness rather increased with decreasing diameter. The silver mirror was sometimes deposited on a flat or conical bottom, so that the incident light should be reflected and its action increased; but very little increase in the deflection of the galvanometer was observed, so that the ordinary spherical shape was chosen.

Efforts have been made to replace the hydrogen by other gases, for instance ethane, ammonia and acetylene. With ethane and the current reversed a very dark violet-blue color was obtained of a high sensitiveness, and of a beautiful metallic luster, but unfortunately the sensitiveness proved not to be constant. When dry ammonia vapor was used instead of hydrogen for the formation, a bright blue layer was obtained of high sensitiveness which however decayed also in the course of time. Acetylene finally formed a black layer with potassium under the influence of the electric field, but it was very little sensitive. So far hydrogen seems to give the most sensitive and the most constant cells.

Four alkali metals have been used, viz., sodium, potassium, rubidium and cæsium. The best results have been obtained with rubidium and neon. The metal was distilled in the cell while the silver mirror was cooled with ice. A potential difference of 280 volts produced a glow in the hydrogen and a very beautiful violet reddish sensitive layer with a bright metallic luster. The hydrogen was then replaced by helium, argon or neon. The neon was received from the Bureau of Standards.

The three curves *A*, *B* and *C*, of Fig. 2 show the relative sensitiveness of the rubidium cells filled with these three gases. Helium gives the smallest, neon the best sensitiveness. Nevertheless it is possible that the helium and argon cells are better than the neon cell because the curve for the neon rises much quicker than the other curves, in other words the neon cell is more sensitive to small changes of the potential difference acting between the electrodes than the helium and argon cell. It is very important to use perfectly pure gases.

The sensitiveness of some cells decays slightly during the first few days after the formation and then becomes constant. Some distinct white spots appeared on the surface of some of the very bright violet rubidium metals, and in one or two instances such a spot became wider

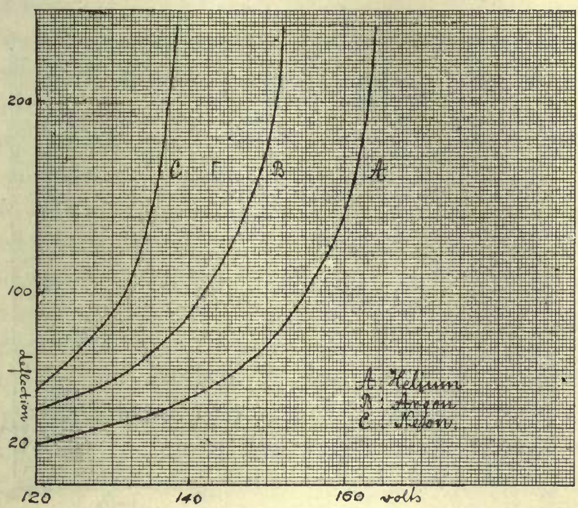


Fig. 2.

in the course of time and covered finally the whole surface, which then appeared bright bluish, and whose sensitiveness was considerably less than that of the original violet surface. When the cells were of a larger size, these bright violet-red surfaces on rubidium were never obtained, but rather sky-blue and blue-green colors which exhibit very beautiful iridescence. The potassium cells were formed with a potential difference of 360 volts. The glow discharge gives almost instantly rise to a most beautiful golden rose color, which is exceedingly sensitive, but not very stable. When the formation is continued for a second or two, a deeper violet-red appears which remains practically constant, but the golden hue gradually fades away. The sensitiveness of the cell when filled with the different gases is shown by the curves of Fig. 3. Neon again

shows the greatest sensitiveness, hydrogen the smallest, argon seems to give a curve which lies between *A* and *B*, but this question is not quite settled. A comparison of Figs. 2 and 3 shows that for rubidium we find the same photoelectric current with a potential difference about 40 volts smaller than for potassium. Very striking iridescence can be obtained by the potassium.

Cells have also been formed with sodium and cæsium. The former metal gives very sensitive cells, but their construction is more difficult than that of the potassium and rubidium cells, the sodium seems to act somewhat on the silver mirror, so that the distilled metal does not seem so bright on the silver as on the glass. If however, the metal is distilled on the glass bulb directly, then the contact with the electrode is unsatisfactory. The pure metal seems to give a very sensitive golden

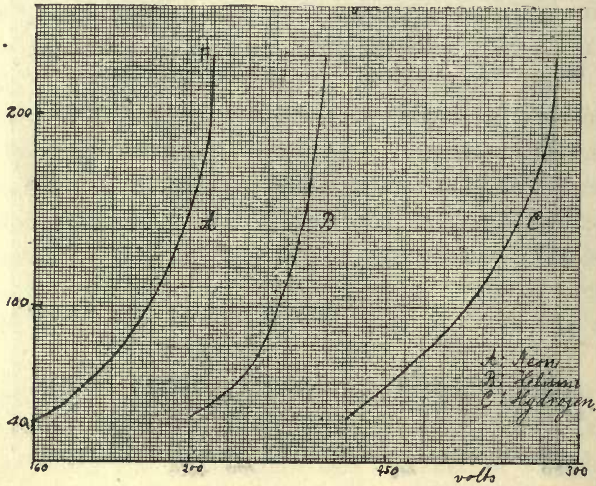


Fig. 3.

layer. The cæsium finally is liquid at 28° and can therefore not be used directly. A solid amalgam of this alkali metal has been formed which was, however, of a rather weak sensitiveness.

The cells described in this article show a very small dark current; if it exists at all, it can be compensated by the application of a convenient small potential at the platinum cylinder between the two electrodes. As far as our present measurements indicate, there is an accurate proportionality between the intensity of the incident light and the photoelectric current. The cells are used in stellar photometry.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
August 12, 1915.

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AN INVESTIGATION OF THE TRANSMISSION, REFLECTION AND ABSORPTION OF SOUND BY DIFFERENT MATERIALS.

BY F. R. WATSON.

THE experiments on the transmission of sound were performed with the following arrangement of apparatus.[1] The source of sound was an adjustable whistle blown by air from a constant pressure tank and mounted at the focus of a specially constructed parabolic reflector with a focal length of nine inches and an aperture of five feet. This was placed in front of an open doorway so that the sound, which proceeded in a large parallel bundle from the reflector, could pass through the doorway into another room. The receiver of sound, a Rayleigh resonator, was mounted in the other room in the path of the sound symmetrically opposite the reflector and doorway and measured the intensity of the transmitted sound.

The resonator used was a modification of Rayleigh's original design.[2] It consisted of a horizontal brass tube closed at one end by an adjustable piston. A mica disc was suspended by a quartz fiber at an angle of 45° with the axis of the tube. When the sound of the whistle reached the resonator it set up a back-and-forth surging of the air in the resonator and caused the mica disc, which was placed at a loop, to rotate. This action is in accordance with the general principle that any flat object in a current of air tends to set itself at right angles to the current. The amount of rotation was measured by means of a lamp and scale in connection with a mirror which was attached to the suspended system above the mica disc.

The readings on the scale are proportional, for small angles of rotation of the disc, to the intensity of the sound. This is shown as follows. The moment M of the couple turning the disc may be proven[3] to be

$$M = kW^2 \sin 2(\theta - \varphi),$$

where W is the velocity of the steam, θ is the angle of repose between the direction of the stream and the normal to the disc, φ is the angle of

[1] PHYS. REV., Vol. V., p. 342, 1915.

[2] Phil. Mag., Vol. 14, p. 186, 1882.

[3] W. König, Wied. Ann., Vol. 43, p. 51, 1891.

deflection, and k is a constant. In case the stream is not steady but alternating, as it would be in the case of the vibrating air in the resonator, W^2 may be replaced[1] by the mean value of W^2. The intensity, I, of the sound setting up the vibrations in the resonator is proportional to the square of the velocity,[2] so that

$$M = k_1 I \sin 2(\theta - \varphi).$$

Finally the turning couple M becomes equal for equilibrium to the restoring couple set up by the twisted quartz fiber and this latter couple is proportional for small angles of rotation to the angle of rotation φ, or

$$M = k_2 \varphi.$$

Comparing the intensities of two sounds we get

$$\frac{I_1}{I_2} = \frac{\varphi_1}{\varphi_2} \cdot \frac{\sin 2(\theta - \varphi_2)}{\sin 2(\theta - \varphi_1)}.$$

In the experimental observations, φ was not measured directly; since the scale readings are proportional to $\tan 2\varphi$, the scale being plane and the angle of deflection being doubled by the reflection of the spot of light from the mirror. Calculations for the data taken show that the ratios

$$\frac{\tan 2\varphi_1}{\tan 2\varphi_2}$$

and

$$\frac{\varphi_1 \sin 2(\theta - \varphi_2)}{\varphi_2 \sin 2(\theta - \varphi_1)}$$

differ about 2 per cent. for the maximum angle of deflection, 11°. Therefore, as stated, the readings on the scale may be taken as proportional to the intensity of the sound.

Measurements were taken, first, through the open doorway, then with one panel of material placed over the doorway, then two panels and finally three panels; the deflection of the resonator being noted for each case. Considerable trouble was experienced in getting steady deflections of the resonator. This was finally overcome to a great extent by arranging a delicate adjustment for keeping constant the flow of air to the whistle, and also by building a small house with a glass window for the observer. Any movement of articles or air in the room changed the deflection of the resonator so that rigid observance of immovability of objects was necessary. The samples to be tested were mounted on similar frames of one-inch cypress strips and fastened over the open

[1] Rayleigh, Th. of Sound, Vol. II., p. 44.

[2] Rayleigh, Th. of Sound, Vol. II., p. 16, and Zernov, Ann. der Phys., Vol. 26, p. 79, 1908.

doorway by two ropes. A strip of hairfelt was mounted around the woodwork of the doorway to prevent sound leaking through at the edges. Preliminary measurements were carried on for some time to get the apparatus and method of taking observations in satisfactory shape. On December 30, two complete sets of observations were taken, the average of these being used to obtain the comparative values of the transmission powers of the different materials. Table I. gives the results obtained.

Table I.

Transmission of Sound.

Material.	Deflection of Resonator in Centimeters.				Average Deflection.			
Thickness in Layers.	0	1	2	3	0	1	2	3
Open doorway	40.3 38.5				39.4			
½″ hairfelt		23.0 22.3	15.3 15.5	10.8 9.9		22.6	15.4	10.4
¼″ cork board		7.7 8.1	3.6 3.9	2.9 2.9		7.9	3.75	2.9
¾″ cork board		1.1 1.2	2.1 2.0	1.0 0.7		1.15	2.05	0.85
¼″ paper-lined hairfelt		5.7 4.3	22.3 21.1	4.4 3.2		5.0	21.7	3.8
¾″ paper-lined hairfelt		7.1 5.9	2.0 1.9	0.5 0.3		6.5	1.95	0.4
¾″ flax board		2.2 2.3	0.5 0.6	0.1 0.1		2.25	0.55	0.1
¼″ pressed fiber		0.4 0.25				0.32		
¾″ pressed fiber		0.2				0.2		

Table II. gives the calculated percentages of the sound transmitted and the sound stopped, it being assumed that the open doorway transmits 100 per cent. or, that it stops 0 per cent.

The data of Table II. are shown in the form of curves in Fig. 1. Inspection of these curves shows that ½ in. hairfelt stops less sound than the other materials, one layer stopping only 43 per cent. Next comes the ¼ in. cork board which stops 80 per cent. for one layer and 90.5 per cent. and 92.6 per cent. for two and three layers respectively. This is followed by the ¾ in. paper covered hairfelt, the ¾ in. flax board and finally the ¼ in. pressed fiber, one layer of which stops practically all the sound. These values do not tell the whole story concerning the acoustical efficiency of the materials, since other qualities must also be considered. The pressed fiber, for instance, is of little value acoustically because its sound absorbing power is very small.

TABLE II.

Transmission of Sound Through Different Materials.

Material.	Percentage of Sound.							
	Transmitted.				Stopped.			
Thickness in Layers.......	0	1	2	3	0	1	2	3
Open doorway.............	100				0			
½″ hairfelt.................		57.0	39.0	26.0		43.0	61.0	74.0
¼″ cork board		20.0	9.5	7.4		80.0	90.5	92.6
¾″ cork board		2.9	5.2	2.2		97.1	94.8	97.8
¼″ paper lined hairfelt.......		13.0	55.0	9.6		87.0	45.0	90.4
¾″ paper lined hairfelt.......		1.7	0.5	0.1		98.3	99.5	99.9
¾″ flax board...............		5.7	0.14	0.02		94.3	99.8	99.9
¼″ pressed fiber............		0.08				99.9		
¾″ pressed fiber............		0.05				99.9		

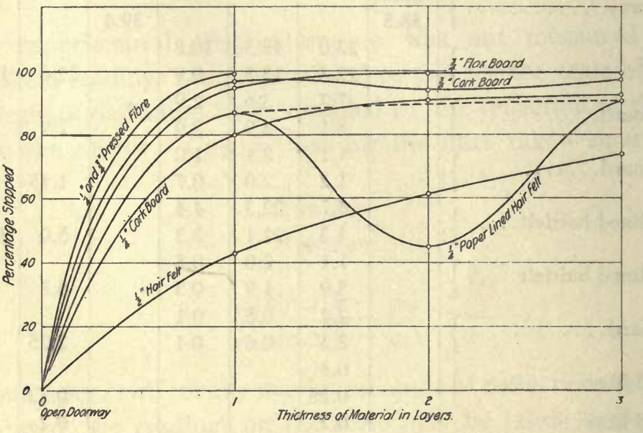

Fig. 1.

Percentage of sound stopped by materials.

Curves showing the percentage of sound stopped by different materials.

The curves for the ¼ in. paper-lined hairfelt show that this sample acts differently than the others. *Two* layers of this material stop *less* sound than *one* layer. Repeated measurements gave the same puzzling result. The ¾ in. cork board shows the same phenomenon, but to a less degree. After some consideration, it was decided to investigate other acoustical properties of the samples to see if additional data would explain this anomalous transmission.

If incident sound falls on a material, three things may happen. The sound may be partly reflected, partly absorbed and the rest transmitted. If these three fractions are added together, they must equal the incident sound, or

$$T + A + R = I = 100 \text{ per cent.}$$

Therefore to know what happens to the incident sound it is necessary to determine the amounts reflected, absorbed and transmitted. On considering the case of the paper-lined hairfelt in the light of this reasoning, it was decided to attempt to measure the reflection of sound.

Reflection of Sound.

By moving the parabolic reflector off to one side, the sound was sent obliquely toward the open doorway where it was reflected by the hairfelt and then passed to the Rayleigh resonator which had been moved into the same room with the reflector and placed so as to be directly in the path of the reflected sound. The observer, as in the transmission tests, stationed himself inside the small house and read the deflection of the

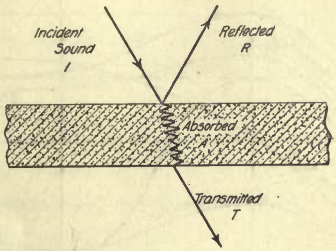

Fig. 2.

Action of a material in reflecting, absorbing and transmitting sound.

resonator through the glass window. A small portion of sound was reflected from the sides of the doorway so that, even with no material over the open door space, the resonator gave a small deflection. This was taken as the zero deflection for the other readings. The deflection for 100 per cent. reflection was arbitrarily taken to be the largest deflection obtained, namely, the deflection given by one layer of ¾ in. cork board. This value is doubtless too small, but probably not much in error, especially when only *comparative* values are being considered.

The average of two sets of observations on the reflection of sound from the materials is given in Table III. and in curves in Fig. 3. The interpretation of the results is best realized by combining curves from Figs. 1 and 3. Thus, for ½ in. hairfelt in Fig. 4, it is seen that the curve for the reflected sound follows very closely the curve for the stopped sound. Consideration of Fig. 2 shows that the sound which is stopped by the material is reflected and absorbed.

The amounts of reflected, absorbed and transmitted sound are thus

TABLE III.

Reflection of Sound by Different Materials.

Material.	Deflection of Resonator in Centimeters.				Percentage of Sound Reflected.			
Thickness in Layers.	0	1	2	3	0	1	2	3
Open doorway	3.9				0			
½″ hairfelt		4.9	6.6	10.5		19	25	40
¼″ cork board		15.7	22.0	22.6		61	85	87
¾″ cork board		25.9	21.2	22.1		100	82	85
¼″ paper lined hairfelt		20.7	5.9	10.0		80	23	39
¾″ paper lined hairfelt		10.4	6.6	9.3		40	25	36
¾″ flax board		22.5	20.0	20.0		87	77	77
¼″ pressed fiber		23.2				90		

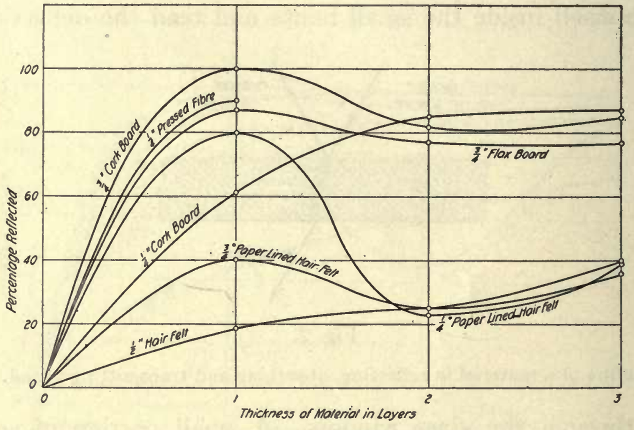

Fig 3.- Percentage of Sound Reflected by Materials

Fig. 3.

Curves showing the percentage of sound reflected by different materials.

easily shown in Fig. 4. The amounts of sound reflected and absorbed increase with the thickness, but the transmission decreases. It should be remembered that the absolute values for the absorption and reflection are doubtless in error but that the comparative values are in correct proportion.

The curves for the ¼ in. paper-lined hairfelt are shown in Fig. 5. The two curves of reflection and transmission follow each other closely. It is interesting to note how the absorption increases uniformly although the transmission and reflection both vary. The probable cause for the anomalous reflection and transmission, as will be discussed later, lies in the vibration of the material due to resonance. Certain thicknesses of the material vibrate vigorously under the action of the sound and thus

create sound waves on the further side of the material. This explanation is advanced also by Weisbach[1] who made a similar test but with different apparatus and method.

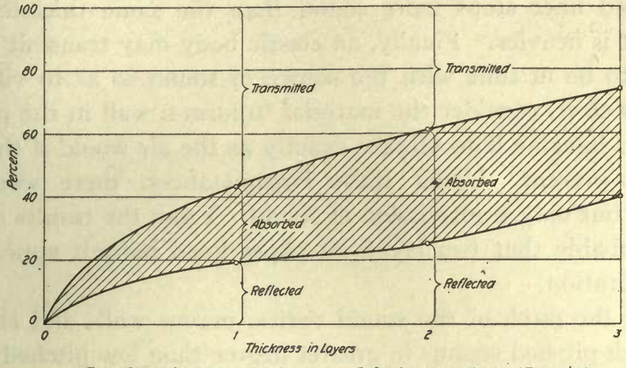

Fig.4. - Curves for Hair Felt, showing how the Reflection, Absorption, and Transmission vary with the Thickness.

Fig. 4.

Curves for hairfelt, showing how the reflection, absorption and transmission vary with the thickness.

Discussion of Results.

The transmission of sound of constant pitch depends on at least three qualities of the transmitting material;—its porosity, density and elasticity. Porous bodies transmit sound in much the same proportion that

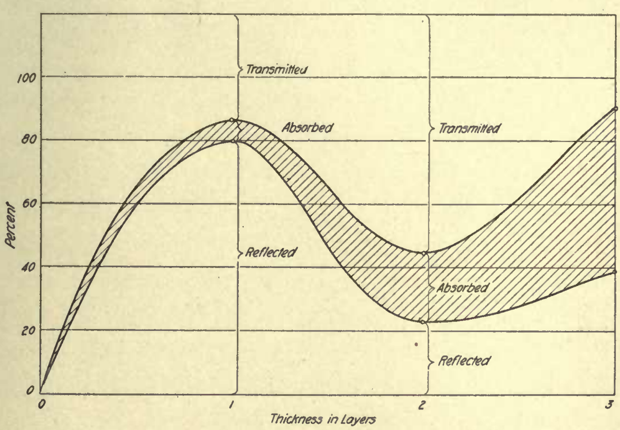

Fig.5 - Showing the Reflection, Absorption, and Transmission of Sound by ¼" paper lined hair felt.

Fig. 5.

Showing the reflection, absorption and transmission of sound by ¼″ paper-lined hairfelt.

[1] "Versuche über Schalldurschlassigkeit, Schallreflexion und Schallabsorption," Annalen der Physik, Vol. 33, p. 763, 1910.

they transmit air.[1] This is why hairfelt transmits more sound than the other samples. Density also plays a part. Two samples stop sound in proportion to their densities, other conditions being equal.[2] Thus the pressed fiber stops more sound than the same thickness of cork because it is heavier. Finally, an elastic body may transmit sound if it happens to be in tune with the source of sound so as to vibrate. To make this clear, consider the material to form a wall in the path of the sound and imagine it to vibrate exactly as the air would if the material were not present. Under these circumstances, there would be no reflection but only transmission of sound. From the results obtained it seems probable that two layers of paper lined hairfelt approximate to such a vibration.

In case the pitch of the sound varies, porous walls and elastic walls reflect high pitched sounds in greater degree than low pitched ones.[3]

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[1] Tufts, Amer. Jour. of Science, Vol. 2, p. 357, 1901.

[2] Jäger, "Zur Theorie des Nachhalls," Sitzber. der Kaiserl. Akad. der Wissenschaften in Wien, Math-naturw. Klasse; Bd. CXX., Abt. II*a*, Mai, 1911.

[3] Jäger, loc. cit.

A STUDY OF RIPPLE WAVE MOTION.

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A STUDY OF RIPPLE WAVE MOTION.

BY F. R. WATSON AND W. A. SHEWHART.

WAVE motion has been made the object of a large number of theoretical and experimental investigations to determine the physical properties of waves and also to explain the various phenomena of reflection, refraction, diffraction and interference. One of the most satisfactory experimental methods of attack has been found in the study of ripple waves, since in this case, we can visualize almost every phenomena of wave motion.

The object of this paper is to describe the development of a method by which ripple waves may be generated not only in steady patterns but also in patterns in which the waves apparently move very slowly. This allows a leisurely examination of the various phenomena. The method also allows the convenient exhibition of the waves to a lecture audience.

Historically, the investigation of ripples began in 1871 when Lord Kelvin[1] observed that the propagation of ripples depended on surface tension. Matthieson[2] tested the validity of Kelvin's formula, but, because of the rough measurements of the waves set up by a pin point piercing a jet of water, failed to obtain a great degree of accuracy. Ahrendt,[3] Riess[4] and others made similar experiments, but it remained for Lord Rayleigh[5] to develop the first accurate method of investigation. To make visible the extremely small disturbances in the plane of the liquid surface, he used a modified form of Foucault's method of testing plane surfaces. Furthermore, he used the stroboscopic method for making the waves appear to stand still. Dorsey[6] and Watson[7] have extended and improved Rayleigh's method in making investigations on the surface tension of liquids.

Tyndall[8] first made use of ripple waves to illustrate wave motion. Later, Vincent[9] was able with more refined apparatus to obtain beautiful photographs of the same phenomena. H. Schultze[10] devised an electrical

[1] Phil. Mag. (4), Vol. 42, p. 375, 1871.
[2] Wied. Ann., Vol. 38, p. 118, 1889.
[3] Exner's Rep. der Physik, Vol. 24, p. 318, 1888.
[4] Exner's Rep. der Physik, Vol. 26, p. 102, 1890.
[5] Lord Rayleigh's Collected Works, Vol. III., p. 383.
[6] PHYS. REV., Vol. 5, p. 173, 1897.
[7] PHYS. REV., Vol. 12, p. 257, 1901.
[8] S. P. Thompson, "Light, Visible and Invisible," Chap. 1.
[9] Phil. Mag., Vol. 43, p. 417; 45, 191; 46, 290.
[10] Zeitsch. f. Instk., p. 151, 1907.

method for producing ripples, modifications of which have been made by Pfund[1] and Palmer.[2] Waetzmann[3] developed a method in which the waves were generated by intermittent puffs of air, and were made visible by flashes of light isoperiodic with the puffs. Waetzmann's method, with extensions and modifications, has been used in the present investigation.

Fig. 1 is a diagram of the apparatus. Ripple waves were generated by puffs of air blown against the water surface in the glass-bottomed tank A. The puffs were secured by cutting a tube conveying compressed air and inserting in the gap a disc with a circular row of equally spaced holes. When the disc rotated, the current of air was periodically interrupted. The waves were made visible by the stroboscopic method. Light from an arc lamp was focused on the row of holes in the rotating disc, thus giving flashes isoperiodic with the puffs of air. By reflecting the light upward through the glass tank, a steady pattern of waves was revealed. Fig. 2 shows a photograph obtained with circular waves.

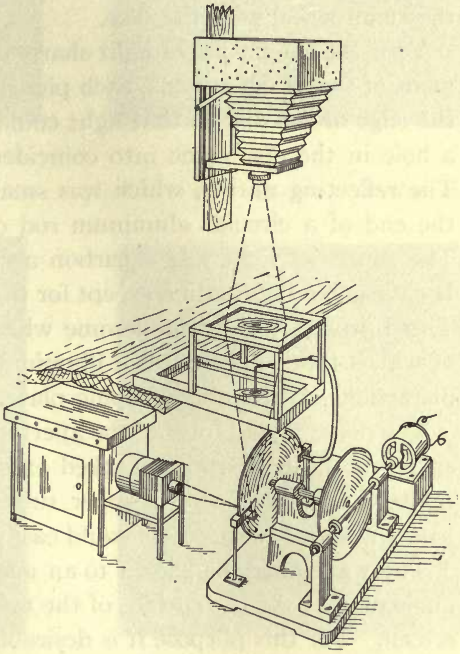

Fig. 1.

Diagram of apparatus showing how ripples are generated on a water surface by puff of air and made visible stroboscopically on the frosted glass plate above the water tank.

By using a second row of holes, which were fewer in number than those in the row already described, the flashes of light could be made to come a little slower than the puffs of air; the result being that the waves apparently moved forward slowly. This action allowed a leisurely study for the different phases of reflection, diffraction, etc., with slowly moving waves.

Trouble was experienced in getting steady patterns of waves, due to vibrations of the apparatus and fluctuations of the puffs of air. The vibrations were overcome by mounting the tank on a steady support. The fluctuations in the air puffs were almost entirely eliminated by using

[1] Phys. Rev., Vol. 32, p. 324, 1911.
[2] Phys. Rev., Vol. 33, p. 528, 1911.
[3] Physikalische Zeitschrift, Vol. 12, p. 866, 1911.

a cast iron disc 10 inches in diameter and one-fourth inch thick and facing it after it had been mounted on an axle so that it would run true. It was then mounted securely on a cast-iron base and rotated by a small wheel on an axle attached by a toggle joint to a 1/6-H.P. direct-current motor. Variation of the speed of the disc was obtained by changing the resistance in series with the motor and also by shifting the position of contact of the small wheel with the disc.

To make the flashes of light sharp and definite in position, an arrangement of two small screens, each pierced by a small hole, was placed over the edge of the disc so that light could pass only during the instant that a hole in the disc came into coincidence with the holes in the screens. The reflecting mirror, which was small in area, was made by polishing the end of a circular aluminum rod cut at an angle of 45° to its axis. The source of light was a carbon arc fed by a 110-volt direct current. It gave good illumination except for the colors explained by Mrs. Ayrton.[1] These proved to be troublesome when taking photographs. A pattern of waves that appeared well illuminated to the eye would show serious distortions on the photographic plate.

The position and form of the aperture for the puffs of air influenced the shape of the waves to a marked extent. Satisfactory results were obtained by using a small copper tube of about 1 mm. diameter placed near the water surface. It could easily be bent into any desired position.

The waves may be shown to an audience by mounting a mirror at an angle of 45° over the surface of the tank so that the light is reflected to a screen. For this purpose it is desirable to let more light through so that the pattern will be well illuminated. This makes the definition of the waves less sharp but still satisfactory enough for demonstration.

Figs. 2 to 7 show some of the patterns investigated. Fig. 4 shows a curious overlapping interference. The patterns in Figs. 5, 6, and 7 were obtained by placing metal forms on the bottom of the glass tank and allowing the water surface to barely cover them. The metal must be free from oil or grease. In Fig. 6 the diffraction waves were made more intense by shielding the central portion from the camera for part of the exposure. The time of exposure varied from 10 seconds to 20 minutes, depending on the amount of light. The length of the waves was measured to be nearly 0.4 cm.

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[1] "The Electric Arc," Chap. 1.

Fig. 2.
Circular waves.

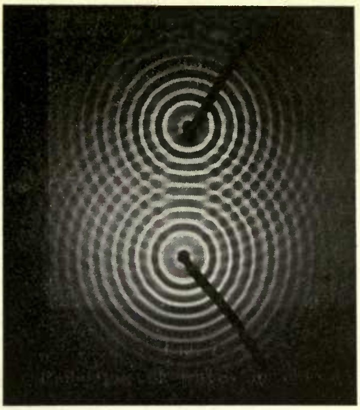

Fig. 3.
Interference of two sets of circular waves.

Fig. 4.
Interference of two sets of circular waves with sources close together.

Fig. 5.
Diffraction through a narrow channel. Also reflection of waves.

Fig. 6.
Diffraction of waves through narrow and wide aperture.

Fig. 7.
Reflection of waves in ellipse. Note the conjugate focus.

F. R. WATSON AND W. A. SHEWART.

[*Reprinted from* SCIENCE, *N. S.*, *Vol. XLIII.*, *No. 1107*, *Pages 393–394*, *March 17*, *1916*]

PHOTOGRAPHS SHOWING THE RELATIVE DEFLECTION OF THE POSITIVE AND OF THE NEGATIVE IONS AS COMPARED WITH THAT OF THE ELECTRON

POSITIVELY and negatively charged ions, atomic in size (commonly called "retrograde rays"), *accompany* the stream of electrons issuing from the cathode in a highly exhausted discharge tube. Thomson[1] studied their properties by placing a photographic plate within the tube in such a position as to receive these rays after being deflected simultaneously by an electric and a magnetic field. When the fields are coincident (not crossed) the displacements on the photographic plate are in directions at right angles to each other. The photographic method is now in common use.

To the writer's knowledge no photographs, however, have been published in which all three of the component carriers—the positive ion, the negative ion and the electron—are shown simultaneously on the same plate. Since the mass of the electron is only 1/1700 that of the hydrogen atom, and since the square of the magnetic deflection varies inversely as the mass, it follows that the electron is driven off the plate by a magnetic field that would give the ion only an appreciably small deflection. By weakening the magnetic field the trace due to the electrons may be retained on the plate.

Two full-sized photographs, Figs. 1 and 2, with key, Fig. 3, are submitted. Comparatively weak magnetic fields were employed.[2]

[1] J. J. Thomson, "Rays of Positive Electricity," pp. 75, 1913.

[2] For arrangement of apparatus see C. T. Knipp, *Phys. Rev.*, Vol. XXXIV., March, 1912.

The two coincident deflecting fields are sketched in Fig. 3, in which the direction of the electrostatic field is indicated by the minus and plus signs, while the arrow heads show

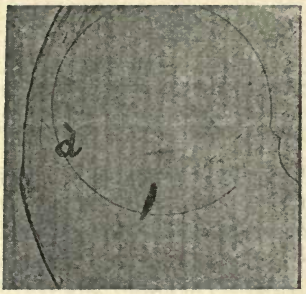

FIG. 1.

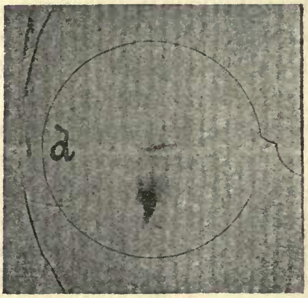

FIG. 2.

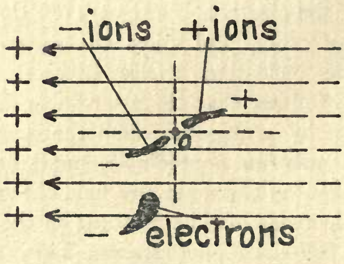

FIG. 3.

the direction of the magnetic field. Again, magnetic deflections are up or down, while electrostatic deflections are to the right or left. The undeflected spot 0 is due to carriers that have lost their charge before entering the de-

flecting fields. In these photographs, Figs. 1 and 2, the traces due to the positive and negative ions unite at the central undeflected spot, the portion to the right of 0 being due to positive ions and that to the left negative ions, while the trace *e, due to electrons,* is distinctly separated from 0 and at some distance from it, and as we should expect, is in the same quadrant as the heavier negative ions. In Fig. 1 the time of exposure was 10 minutes, electrostatic field 2,070 volts per centimeter, magnetic field 1.7 amperes, and the vacuum .011 mm. mercury; while in Fig. 2 the corresponding values were 20 min., 2,070 volts, 2.25 amperes, and .005 mm. mercury. The effect of the stronger magnetic field is distinctly shown in Fig. 2 by the increased displacement from 0 of the trace due to the electrons.

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[*Reprinted from* SCIENCE, *N. S., Vol. XLIII., No. 1118, Pages 787–789, June 2, 1916*]

ELECTRICAL DISCHARGE BETWEEN CONCENTRIC CYLINDRICAL ELECTRODES

IN operating vacuum tubes we invariably use an induction coil or an electrostatic machine. The discharge in either case is never quite steady and hence these methods of operation do not lend themselves well to a critical study of the growth of the cathode dark spaces. A steady, and of course continuous, discharge may be had if the current is drawn from a high potential storage battery. Ordinarily it takes more cells than are available; however, by a right choice of conditions a rather extended study may be made with direct current potentials of less than 1,000 volts. The following experiments with concentric cylindrical electrodes were performed recently by the writer in class demonstration.

The discharge vessel consists of an ordinary three-quart battery jar. A hole bored through the bottom receives the evacuating tube, the junction being made airtight with ordinary sealing wax. The lip of the jar is ground flat to receive the plate glass lid. The junction here is made by means of the frequently used half-and-half wax, beeswax and resin. This wax because of its low melting point admits of easy removal of the glass plate. The electrodes are concentric cylinders and may well be made of sheet aluminum—one electrode to fit snugly the inner wall of the jar, and the other mounted on a cylinder of glass tubing about 1½ inches in diameter, which in turn is supported accurately concentric by sealing wax from the bottom of the jar. Outside connections to the electrodes are made by fine bare copper wire run out through the waxed

joints. The assembled discharge vessel is shown at *a* in Fig. 1.

The vessel may be exhausted by a Gaede mercury or a Gaede piston pump and, if desired, the vacuum carried farther by the use of charcoal and liquid air, though the latter is not necessary. The potential employed by the writer to produce the discharge was furnished by a cabinet of high potential storage cells of 1,000 volts.

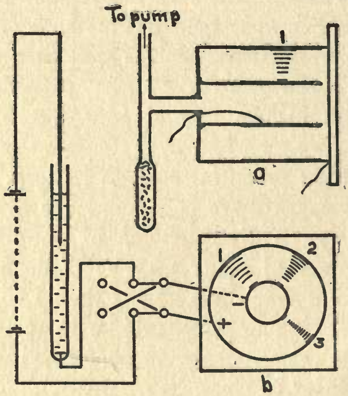

FIG. 1.

Two methods of operating were employed. In the first an adjustable water resistance is connected in series with the cells and discharge vessel as shown at *b* in Fig. 1. When the vacuum is right a beautiful discharge will make its appearance as patches of light on the electrodes. These patches of light, when there is considerable resistance in the circuit and the vacuum is not very high, will be opposite each other and the discharge, as a whole, will wander about, sometimes swinging entirely around, or at times travelling to the edges of the electrodes, only to break away and move to some other point. The movement of the cathode glow (which is the smaller and hence the brighter) is similar to that of the cathode star over the surface of mercury in a mercury

vapor lamp. These areas grow as the vacuum improves when ultimately the entire surface of each electrode is covered. Or, with the vacuum kept constant, the areas may be made to increase in size by cutting out resistance. Hence by improving the vacuum and at the same time cutting out resistance the discharge, if the inner cylinder is made cathode, grows rapidly into a brilliant bull's-eye. The appearance is very realistic, for if now resistance is cut in, the dark space around the cathode (as is evident after a moment's reflection) grows smaller, and *vice versa.* Its

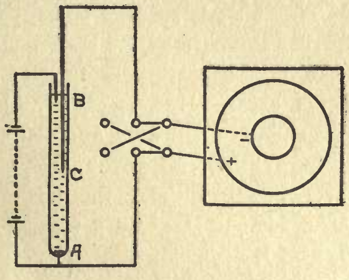

FIG. 2.

outline is exceedingly sharp and perfectly steady, and yet, though the discharge appears very brilliant, the current required may not exceed 20 milliamperes.

This form of discharge vessel offers an interesting method for the study of the striations and their relative spacing with reference to the impressed discharge potentials. These effects are best shown when the vacuum is not too high and the discharge potential is adjusted to give a patch on the cathode, which we will take as the inner cylinder, of about one square centimeter in area. Under these conditions the Faraday dark space should be about 8 mm. in length, and the Crookes dark space should be just visible between the velvety cathode glow and the cathode electrode.

Another prerequisite is that the discharge must not cling to the edge of the aluminum electrodes, but should occupy some intermediate position as shown at 1 in *a,* Fig. 1. In this position the characteristics of the discharge are shown with exceeding clearness. If now some additional resistance is cut in, the area of the discharge will become less, the Faraday dark space will shorten, the positive column will move towards the cathode, and the number of striæ in it will increase, the extra striæ being, as it were, drawn out of the anode. The configuration is perfectly steady except that the discharge, as a whole, is liable to wander. This transition may be continued by a still further increase of the resistance in the circuit, the dark space becoming ever shorter, the positive column lengthening and at the same time shrinking in area and the striæ increasing in number, all without loss of outline or brightness. Finally, the discharge will cease. The various stages are suggested at 1, 2, 3 in *b,* Fig. 1.

In the second method the discharge vessel with its commutator is placed in a derived circuit (Fig. 2). This arrangement enables the discharge potential to be continuously varied over a wide range, and hence for a given vacuum the relation between the length of the dark space and the impressed voltage may be exhibited. Again this arrangement enables the minimum potential to be readily determined that will maintain a discharge. As an example, for a given vacuum with the resistance AC equal to 1/3 that of AB the discharge was observed to just pass, indicating that the potential necessary was 330 volts.

Additional phases of the experiment will suggest themselves to the operator.

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March 4, 1916

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[Reprinted from the PHYSICAL REVIEW, N.S., Vol. VII, No. 6, June, 1916.]

RETROGRADE RAYS FROM THE COLD CATHODE.

BY ORRIN H. SMITH.

J. J. THOMSON,[1] as early as 1897, showed that a system of rays of an entirely different character from the cathode rays accompanies the cathode beam. He found that these rays proceed normally from the face of the cathode, that they are not appreciably deflected by a permanent magnet, and that they possess very little, if any, power of producing phosphorescence.

In 1906 Villard[2] gave an account before the French Academy of the rays accompanying the cathode beam which are not so readily deflected as the cathode beam but which were deflected in such a direction and by such an amount as would be expected of the "kanal strahlen." He noticed that in a mixture of oxygen and hydrogen (or water vapor) the cathode rays produced a luminescence characteristic of oxygen, but when these were deflected aside by a magnet there remained rays which produced a luminescence characteristic of hydrogen. He explained their presence by saying that they were the positive canal rays which fall against the cathode and rebound. To explain their rebounding beyond the limits of the cathode dark space, he assumed that the potential fall underwent rapid variations or was even discontinuous. A stroboscopic test showed this to be true; however, this was to be expected since he used a transformer to produce the discharge.

The following year Thomson[3] showed, independently, that these rays were deflected by strong electric and magnetic fields and that they possessed considerable mass. In a later work he observed that they were very feeble under the most favorable conditions of vacuum, discharge potential, etc., and were exceedingly feeble when the gas pressure in the discharge tube was very low. In this latter respect they were quite different from canal or positive rays. Employing a tube having an opening of about .5 mm. in diameter he obtained a photograph which showed that these rays contain (*a*) positively electrified atoms and molecules of hydrogen, (*b*) positively electrified atoms of oxygen, and (*c*) negatively electrified atoms of hydrogen and oxygen. The photograph

[1] Proc. Camb. Phil. Soc., IX., p. 243, 1897.
[2] Comptes Rendus, CXLIII., p. 673, 1906.
[3] Phil. Mag., XIV., p. 359, 1907.

showed the intensity of the lines corresponding to the negative ions to be greater than that of the positive ions. With the ordinary positive rays the positive lines are the more intense.

The conditions under which retrograde rays are produced are quite different from those that obtain for the ordinary positive rays and for this reason it seemed worth while to repeat and extend Thomson's investigations.

Thomson does not find the molecule of oxygen with the negative charge while in this investigation the molecule of oxygen and the molecule of hydrogen are the only carriers obtained with a negative charge, no atoms appearing at all. The presence of helium in the discharge chamber apparently makes no difference in the photographic result.

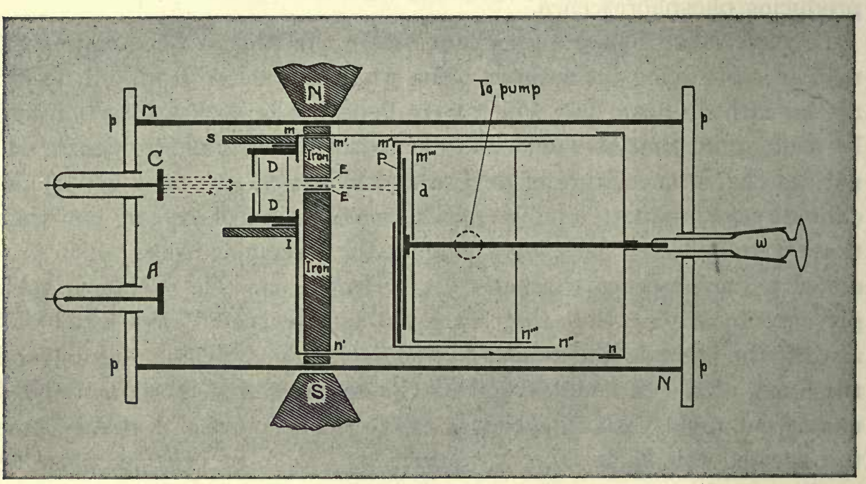

Fig. 1.

Top View. *MN*, containing vessel of glass; *pp*, glass end plates; *mn*, large brass cylinder; *m'n'*, magnetic field extensions; *EE*, electrostatic field plates, connections to which are not shown; *m''n''*, plateholder; *P*, photographic plate mounted on disc *d*, supported by telescoping cap *m'''n'''*, and turned by winch *w*; *DD*, aluminum diaphragms; and *SI*. iron shield.

It appears that Thomson was unable to use a tube of less than .5 mm. bore, while in this investigation traces were obtained with a tube and set of diaphragms having openings of about .05 mm. thus producing sharp lines on the plate which made possible more accurate measurements.

Owing to the short range at which these rays were obtained on the photographic plate, the increased sharpness of the lines, and the restricted range of their velocities due to a restricted cathode dark space, it was possible to obtain some evidence on the question as to whether the power of a particle to affect a photographic plate is a function of its

velocity, momentum, or kinetic energy. This evidence seems to indicate that it is a function of the kinetic energy and that the mean value is about 7.4×10^{-9} ergs.

The apparatus, shown in Fig. 1, and the manipulation is essentially the same as that described by Knipp[1] except that a cold cathode was used instead of the Wehnelt cathode and the discharge was produced by an induction coil. The cathode was just like the anode and similarly placed facing the line of the tube and the diaphragms.

A large Leeds induction coil was operated on ten storage cells. The vacuum was maintained with the aid of a large charcoal bulb dipping into liquid air. In general the vacuum improved with sparking. After a few runs it was found that the liquid air could be removed after about ten minutes from starting, and, as the sparking and pumping continued, it could be dispensed with altogether. Finally the vacuum was so easily maintained that it was necessary to keep the pump itself turned off for about three fourths of the time.

There is a point of interest in connection with the charcoal bulb. It was left on the apparatus for weeks after its use was found unnecessary, remaining all the while at the nearly constant room temperature. The pumps were unable to produce a vacuum of .005 mm. in fully three hours' time when starting from atmospheric pressure, and this was the case whether the bulb was heated for an hour during that time or not. However, if it was pumped to a pressure of one or two mm. and left to stand for ten to fifteen hours then upon starting the pumps a vacuum of .005 mm. could be attained in twenty to thirty minutes. This, strangely, was true even when the vacuum had been let down for a very few minutes and then the pumps started again immediately.

The photographic plate used was Seed's Yellow Label lantern slide plate. This plate is very slow and hence produces great contrast which is the thing desired. Thomson[2] points out that the large ions affect only the surface of the film and do not penetrate like the faster moving electrons, into the film. Hence the plate best suited for this work is one that is slow and that has a thin film with a high percentage of silver. The best traces that could be gotten in this investigation were in many instances so thin that they could hardly be seen. They were obscured easily by the slightest fogging. For this reason a fast plate could not be used. Seed's Gilt Edge Number Twenty-seven plate was tried but in every instance fogging obscured the lines. The Double Coated Cramer Crown plate was tried and found to be entirely too sensitive. Some

[1] Phys. Rev., XXXIV., p. 215, March, 1912.

[2] Thomson, Rays of Positive Electricity, p. 4.

experience by another member of the department obviated the necessity of trying the Cramer X-ray plate. As an instance to show that these carriers affect only the surface of the film, the author gently stroked the film under water with a fine camel's hair brush to remove foreign particles and it was found that, in some cases, the lines were entirely obliterated. Further, after the negative had dried the lines could be obliterated by breathing on the film and wiping it gently with a soft cloth. In both cases, other than erasing the lines, no further change could be detected in the film. It was found advantageous to put some alum in the fixing bath to harden the film. The developer used was ordinary hydrochinon, the time of development being from six to twelve minutes.

The time of exposure varied from thirty minutes for the small to three hours for the larger deflections. There seems to be a limit to the intensity that is obtainable, for after a certain length of exposure the intensity of the lines did not apparently increase with further exposure. This was true for long or short development or even when they were exceedingly dim. This is in agreement, however, with the theory that they affect only the surface of the film.

Thomson found that the retrograde rays were best obtained when the gas pressure was not too low. The present photographs bear out that fact very well. If the vacuum was kept about .002 to .004 mm. scarcely any trace of the rays could be found on the plate. The best pressure for their production seems, from this investigation, to be between .015 and .008 mm.

There is always a central spot that is undeflected which is probably due to neutral carriers that were negative originally but which lost one electron before they got into the deflecting fields. It would seem from this that a moving particle need not be charged in order to affect a photographic plate. It is quite evident that the velocity of an uncharged particle must be above a certain value otherwise a plate would be affected by exposure to the air in a dark room due to no other agency than to the velocity of the air molecules produced by ordinary heat agitation. The mean of this velocity at 0° C. for the hydrogen molecule is about 2×10^5 cm./sec. and for the oxygen molecule about 4.5×10^4 cm./sec. Whether the ability of a moving particle to affect a photographic plate is due to its momentum or its kinetic energy, or simply to its velocity, is not definitely known. It seems reasonable to expect, however, that it should be a function of one of these. On a number of the plates the lines were distinct enough to locate approximately the place where the slowest ions would strike, *i. e.*, those that had just sufficient velocity to affect the plate. These points were found, in every

case, to be well within the limits of the field, *i. e.*, so far as the limits of the apparatus are concerned the lines might have extended farther from the origin. It occurred to the author then to assume that there were particles which struck beyond the last points of the visible trace but whose velocity was not sufficient to cause them to affect the film. If the coördinates of the last visible point in each line be measured and v and e/m determined, then, for all such points, we should get a constant, showing whether this minimum effect on the plate is a function of the velocity, the momentum, or of the kinetic energy of the moving ion. Table I. shows values which are proportional to the velocity, momentum, and kinetic energy for the points in question on sixteen different lines. It can be seen that the values for the kinetic energy are nearly constant while the values for the velocity and the momentum are not constant. It thus appears that the power of a particle to affect a photographic film probably depends on its kinetic energy. The mean of these values of the kinetic energy is, from Table I., 7.4×10^{-9} ergs which is the minimum required. This value would probably be different for an electron because of its size. It is somewhat larger than the energy required to produce an ion which is 1.63×10^{-11} ergs. The above value (7.4×10^{-9}) was calculated from data obtained from this investigation, except for the value of e, by the formula

$$\text{kinetic energy} = 1/2 \cdot m/e \cdot e \cdot v^2.$$

The value of e was taken as 1.55×10^{-20}.

Table I.

Photographic Plate.	Line.	Constant × Velocity.	Constant × Momentum.	Constant × Kinetic Energy.
75	Upper	6.04	18.36	111.0
76	Upper	6.54	17.99	117.6
76	Lower	1.36	87.18	118.6
85	Upper	6.37	12.50	79.0
85	Lower	1.52	52.50	79.7
86	Upper	6.23	12.71	79.2
86	Lower	1.53	52.48	80.4
87	Upper	7.27	19.40	102.5
87	Lower	1.69	60.50	102.3
88	Lower	1.37	61.70	84.5
94	Upper	5.55	10.71	106.4
94	Lower	1.77	36.28	100.4
95	Upper	7.41	15.03	91.1
95	Lower	1.64	51.50	84.47
96	Upper	6.35	17.70	112.5
96	Lower	1.42	59.36	84.26

It can be seen from Table I. that, even though the values of the kinetic energy vary somewhat, the values for a given plate as a rule are more nearly alike. Plate ninety-six furnishes the greatest variation from this rule. It might be reasonable to expect that different emulsion numbers would reveal slightly different kinetic energies required to affect the film. Several emulsion numbers are represented in these data.

The photographs taken with the apparatus in the last refinement, while clear and capable of accurate measurement, do not lend themselves to reproduction and hence are omitted. The important dimensions are as follows:

Length of electrostatic field	1.10 cm.
Length of magnetic field	1.10 cm.
Distance from point of emergence to plate	1.58 cm.
Length of triangular test coil[1]	2.52 cm.
Base	.63 cm.
Number of turns	19.

The negative lines show distinctly the parabolic heads which are not in evidence on the positive lines. It was evident from nearly all the plates exposed that the negative carriers are in preponderance over the positive ones. This seems reasonable to expect since the distance to the plate is, for the lower pressures, within the limits of the mean free path and it is necessary to assume that every positive carrier has lost two electrons between the outer limits of the dark space and the deflecting fields. If this is true we should expect that the lines due to the positive carriers would not be as sharp as those due to the negative carriers, the ions being deflected somewhat from their true path in the process of losing an electron. Most of the photographs bear this out. It is somewhat surprising, in consideration of the foregoing, that this preponderance is not greater than the photographs seem to indicate unless the negative ion is more unstable than the positive ion. An additional suggestion in the same line comes from a study of Thomson's photographs of positive rays, in a great many of which the negative counterpart is very weak or cannot be seen at all on the prints when the positive lines are very pronounced. The positive lines do not have the distinct parabolic head that the negative lines have. They are also broader and more diffuse. Joining the parabolic head to the center is a line due to the secondary rays of Thomson. This is shown particularly in one exposure where the electric field overlapped the magnetic so that the secondary line does not join straight on to the head of the parabola. The data for exposure number eighty-five, are given in Table II. This

[1] Thomson, Rays of Positive Electricity, p. 10.

indicates that the carriers which produced the two lines are the molecules of hydrogen and oxygen respectively. The measurements of the coördinates were made with an ordinator composed of a frame to which the plates could be fastened so that there was a movable point above the plate capable of being carried in either of two directions perpendicular to each other by micrometer screws. A Grassot fluxmeter was used to determine the strength of the magnetic field.

Table II.

Photographic Plate Number 85.

Measurements for the Upper Line.

Position.	z in mm.	y in mm.	$v \times 10^{-7}$ cm./sec.	$e/m \times 10^{-4}$	Electric Atomic Weight.	Carrier.
1	3.34	6.42	6.37	.509	1.97	H_2
2	2.46	5.48	7.39	.504	1.99	"
3	1.88	4.77	8.96	.500	2.00	"
4	1.12	3.51	10.38	.458	2.18	"

Measurements for the Lower Line.

Position.	z in mm.	y in mm.	$v \times 10^{-7}$ cm./sec.	$e/m \times 10^{-4}$	Electric Atomic Weight.	Carrier.
1	3.34	1.53	1.52	.029	34.5	O_2
2	2.46	1.31	1.77	.0288	34.8	"
3	1.88	1.16	2.04	.0296	33.8	"
4	1.12	.84	2.50	.0260	38.5	"

Time of exposure, 3.25 hours.
Gas pressure varied between .008 and .018 mm.
Electric deflecting field, 965 volts.
Magnet current, 4.25 amperes.
$A = 8{,}040$, $B = 267 \times 10^9$

All the photographs were exposed with residual air in the discharge chamber except number 88. In this instance it contained some helium but no traces appear in the photograph, in fact in no case does anything appear in any of the photographs except the lines due to the molecules of hydrogen and oxygen. In some cases the positive rays are not visible.

The data show very well how the velocity varies for the carriers striking at the various points along the parabola, that it decreases with increase of distance from the undeflected spot. The value of v and e/m obtained for the smaller values of the electric field are in general less reliable than for those for which the deflection is larger. The "electric atomic weight" of a carrier Thomson[1] has defined as the ratio of m/e for that carrier to m/e for the atom of hydrogen.

[1] Phil. Mag., XXI., p. 234, Feb., 1911.

It was noticed in connection with these experiments that the discharge in the chamber passed more easily with the presence of a transverse magnetic field. Earhart[1] has shown that this is true for a longitudinal field.

Summary of Conclusions.

The results of this investigation may be summarized briefly as follows:

1. When obtaining retrograde rays in residual air the molecule of hydrogen appears on every plate accompanied by a heavier carrier which in most cases is the molecule of oxygen. The velocities obtained by the author are smaller than those obtained by Thomson. This is due to the position of the cathode with reference to the small canal through which the carriers pass, the dark space extending beyond the near end of this tube and hence the carriers not attaining their maximum velocity.

2. The negative lines are clearer and sharper than the positive; probably because of the disturbance to the path of the positive particles in the process of becoming positive.

3. Retrograde rays can be obtained with a canal having a bore of about .05 mm. diameter. The best range of pressures for their production is between .008 and .015 mm. of mercury.

4. The power of a moving particle to affect a photographic plate seems to be a function of its kinetic energy. The minimum required for the heavy carriers is of the order 7.4×10^{-9} ergs, which is larger than the energy required to produce an ion, however, there is evidence in favor of the view that this value may depend somewhat on the emulsion on the plate.

In conclusion I wish to express my thanks to Professor A. P. Carman for the excellent facilities placed at my disposal and to Dr. C. T. Knipp for his interest and help in carrying on the investigation.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS.

[1] PHYS. REV., Feb., 1914.

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AN EXPERIMENTAL VERIFICATION OF THE LAW OF VARIATION OF MASS WITH VELOCITY FOR CATHODE RAYS.

BY LLOYD T. JONES.

INTRODUCTION.

THE mass of the electron has been shown by W. Kaufmann[1] to be electromagnetic in origin. This mass is shown by the elementary laws of electromagnetism to be constant for small velocities of the electron. M. Abraham[2] has developed an electro-dynamic theory of moving electrons by which he accounts for the falling off of the ratio e/m for electrons moving with high velocities. If β is the ratio of the velocity of the electron to that of light, the ratio of the mass of the electron moving with the velocity v to its mass, m_0, when moving with a slow velocity is

$$\frac{m}{m_0} = \frac{3}{4\beta^2}\left[\frac{1+\beta^2}{2\beta}\log\frac{1+\beta}{1-\beta} - 1\right].$$

The Lorentz-Einstein formula, which satisfies the principle of relativity, gives the ratio of the masses as

$$\frac{m}{m_0} = [1-\beta^2]^{-\frac{1}{2}}.$$

Kunz[3] has discussed the bearing of these formulæ in connection with an electromagnetic emission theory of light and has developed three forms of the formula based on possible changes of form of the electron.

Stark[4] has found that the mass of the cathode particle increases as the velocity increases. The maximum velocity employed by him, 1.14×10^{10} cm. per sec., was, however, not great enough to cause more than a small per cent. increase in the mass.

Later Guye and Ratnsoky[5] carried out an experiment employing rays of 14.7×10^{10} cm. per sec. velocity and obtained an increase of nearly twenty per cent. in the mass.

Each of the investigators has employed a method in which the cathode

[1] W. Kaufmann, Gott. Nachr., 1901, Heft 2; 1902, Heft 5; Phys. Zeitschr., 4, 54, 1902.
[2] M. Abraham, Gott. Nachr., 1902, Heft. 1.
[3] J. Kunz, Arch. des Sci., Jan. 1913; PHYS. REV., p. 464, 1914.
[4] H. Stark, Verh. d. Deut. Phys. Gesell., 5, p. 241, 1903.
[5] Guye and Ratnosky, Arch. des Sci., 31, p. 293, 1911. Guye and Lavanchy, Comptes Rendus, p. 52, July, 1915.

beam traverses nonuniform electric and magnetic fields and the deflection is shown on a phosphorescent screen placed perpendicular to the path. This necessitated a homogeneous cathode beam.

The conclusions of the experimenter must be based on a large number of observations taken at each of a number of different velocities. The method that has been developed for the present research lessens materially the difficulties encountered in a verification by cathode rays and is applicable equally well for the β particles of radium. Perhaps the best feature of the method is that it is desired to have rays of all possible velocities present in the discharge rather than a homogeneous beam. This allows one to use the discharge from a high potential transformer without any additional pieces of apparatus to operate during the time of exposure. Since from a single photograph calculations may be made of e/m for all the velocities present it is possible to obtain the desired results by a single exposure.

The Apparatus.

In a previous determination of e/m and v for cathode rays[1] an apparatus was used involving the same principles as this; the high discharge potentials used in the present investigation, however, necessitated a change in the manner of introducing the electrodes and more effective insulation guarding against the ionizing and direct effect of the discharge.

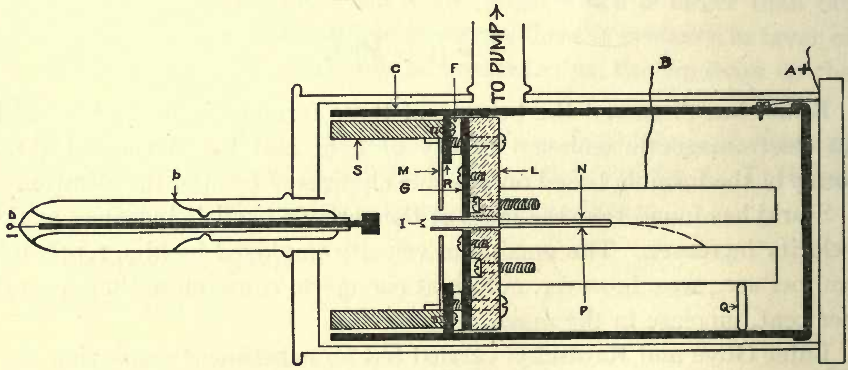

Fig. 1.

A glass jar 11.5 cm. in diameter and about 35 cm. long had a 2.2 cm. hole bored in its base through which the cathode was introduced. The cathode was an aluminum disc about .8 cm. in diameter carried on a small brass rod encased in a small glass tube and connected with one terminal of the transformer through a platinum wire sealed in the glass.

[1] L. T. Jones, Phys. Rev., N.S., Vol. III., p. 317, 1914.

The glass tube encasing the cathode rod was supported at two places by a second glass tube sealed, as shown at *b* in Fig. 1, to a tube of 2 cm. diameter which passed through the hole in the base of the jar. This tube was fastened to the base by sealing wax. A brass cylinder, *C*, of 10.2 cm. diameter and about 32 cm. long was fastened rigidly (manner not shown) to the glass jar; and a brass ring, *R*, of 1.5 cm. width and .4 cm. thickness was soldered inside it with its plane perpendicular to the axis of the brass cylinder. An iron cylinder, *S*, of 7.5 cm. inside diameter and .8 cm. thickness was fastened by screws to the ring *R*. An ebonite ring, *F*, of nearly the same dimensions as *R* was fastened to *R* by screws whose heads were sunk well beneath the surface next *G*. *G* was a brass disc of .3 cm. thickness fastened by brass screws to the ebonite disc, *L*, which was of .8 cm. thickness and carried the electrostatic plates. To increase the insulation discs of mica were placed between *G* and *L*, *F* and *G* and *R* and *F*. To prevent trouble due to the heavy discharge a brass disc, *M*, was held against *R* by a slip ring in *S*. A few millimeters' space was left around the small iron tube, *I*, which passed through directly in front of the cathode.

The two electrostatic plates were brass plates $20.5 \times 7.5 \times 1.2$ cm. In the upper one was inlaid a piece of soft iron, *N*, $5.13 \times 1.4 \times .1$ cm. A similar piece of iron, *P*, was held against *N* by eight short iron screws. After the iron piece, *N*, was inlaid and all necessary holes had been made in the plates they were annealed and then one side of each was surfaced to within .001 cm. of plane. The slip *P* also had its face next *N* made plane. A scratch of about .05 cm. width was drawn full length on the plane side of *P*. The ends of this scratch, for about 1 mm. of their length, were closed with solder and the solder cut off flush with the surface. A small cut was then made in each of the bits of solder and these cuts determined the path of the electron immediately before its entrance into the deflecting fields. The electron then takes the path indicated by the dotted line in Fig. 1. The electron is thus protected from the fields until it leaves the constricting canal. Care was taken that the small cut marking the entrance of the electron in the fields was perfect to the ends of *N* and *P* and that the ends of *N* and *P* were exactly even. As a final precaution a small bit of solder was placed in the middle of the canal as well and a small cut made in it. This insured a straight beam through the tube. Each of these cuts was .01 cm. in width and of about the same depth. The softest iron obtainable was used throughout and the brass was free of magnetic material.

The ebonite disc, *L*, with its plate, *G*, was held against the ring, *F*, by four heavy brass screws threaded into *R*. They were insulated from *G* by an air space of about 2 mm.

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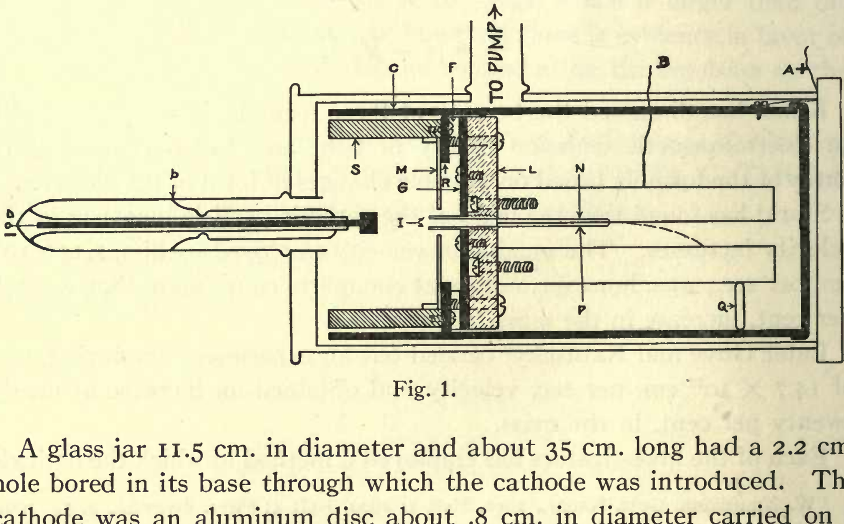

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The two electrostatic plates were held at a fixed distance apart by four porcelain blocks placed one at each corner. Under the back end of the lower plate was placed a brass leg, Q, of adjustable height which served as an additional support for the plates and at the same time connected the lower plate electrically with the brass cylinder, and through it with M and S.

A hole 2.2 cm. in diameter was bored in the side of the glass jar at a suitable position and a glass tube waxed to the jar here connected the vessel to the molecular pump. A wire, A, was connected to the brass cylinder near the back where it could be easily reached from the outside. A was connected to earth and to the second terminal of the transformer. The surfaces of M and S served as anode. The upper of the electrostatic plates was connected electrically to the outside by a wire, B, passing through an insulating plug in the brass cylinder and through a small hole in the glass cylinder. The hole was made vacuum tight by sealing wax.

The photographic chamber was made light tight by closing the ends of the cylinder with a brass cap and the jar was made vacuum tight by closing with a glass plate sealed on with a mixture of beeswax and resin.

The electrostatic potential was applied to A and B and the transformer connected to A and D.

The Spacing Blocks.

Each of the four spacing blocks placed between the corners of the electrostatic plates was a length of a porcelain tube of 1.2 cm. external diameter. After the sections had been cut from the tube they were waxed inside a short piece of brass tubing whose outside was accurately round so it could be chucked in the grinding machine. They were ground down till they were of nearly the same length and the end planes parallel. They were then finished by hand on an iron plate with emery till they were very accurately the same length and the end planes parallel as the measurements showed. The length of the blocks was measured by an optical lever of 24.415 cm. length with a scale 3 meters distant. The lengths of the blocks were $.9822 \pm .0008$ cm.

The Electrostatic Potential.

A high potential storage battery, T, was used to send a small current through two high resistances, M and R, shown in Fig. 2. M consisted of two Wolff boxes aggregating 2×10^6 ohms and R was an adjustable resistance of 10^4 ohms. The potential drop across a part of M was compared by means of the potentiometer, P, with a Weston standard cell of 1.0183 volts at 24° C. The standard cell checked with one recently received

from the Bureau of Standards. By adjusting R the value was easily kept constant to within 1 volt and the value thus determined to less than .1 per cent. The two electrostatic plates were connected directly to the terminals of M.

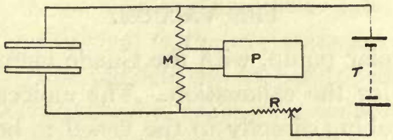

Fig. 2.

The Magnetic Field.

The magnetic field was due to a constant current through 240 turns of wire wound on a rectangular wooden frame about 160 × 60 cm. The cross-section of the coil of wire was about 2 × 2 cm. Calculation showed the field to be uniform over a range greater than that used.

The field was calibrated by the aid of a solenoid of 1,141 turns and 149.83 cm. length wound uniformly on a wooden frame of about 6 × 9 cm. cross-section. The solenoid was placed in the geometrical center of the rectangular frame so that the fields either coincided or opposed each other. A small coil of about 200 turns of very fine wire wound on an ebonite rectangle 2 × 8 cm. was then placed in the center of the solenoid. This coil was connected to a Grassot fluxmeter whose scale was about 4 meters distant. A known constant current was sent through the coil to be calibrated and the current through the solenoid adjusted until the fluxmeter showed no deflection when the two currents were broken simultaneously. The ratio of the currents, 70 to 13.55, gave the value of the field to be 1.854 gausses per ampere. A field of .002 gauss produced a deflection of .3 mm. on the fluxmeter scale.

The Measurement of the Current.

The current in the magnetic field was measured with a Siemens & Halske ammeter of 150 scale divisions which, with the shunt used, had a range of 0 to 3 amperes. The ammeter was calibrated by passing a current through it in series with two Hartmann & Braun standard resistances of .1 and 1 ohm. The potential drop across each resistance was measured by the Wolff potentiometer against the Weston standard cell and the current calculated. The standard resistances were kept in an oil bath at constant temperature. The Reichsanstalt certificates showed the resistances to be sufficiently correct. The calibrations by the two resistances checked. Throughout the calibrations and experiment an adjustable resistance was used to set the ammeter needle exactly on a

scale mark in order that any variation in the current could be more easily detected. With special care taken for good contacts little difficulty was experienced in keeping the ammeter needle exactly on the division mark.

The Vacuum.

The Gaede molecular pump, with the Gaede mercury pump as a preliminary, was used for the exhaustion. The molecular pump was connected by 30 mm. tubing directly to the vessel to be exhausted with no stopcock or other constriction intervening. The mercury pump was connected to a McLeod gauge and a large tube of cocoanut charcoal. The order of starting the pumps assures freedom of mercury vapor in the discharge tube. The construction of the apparatus with its sealing-wax joints made it quite impossible to heat the vessel to rid it of moisture. Such a proceeding proved unnecessary with the wide connecting tubes used, however, as an hour of pumping was usually sufficient to produce a vacuum that caused the transformer to spark 20 cm. between its point terminals rather than pass through the discharge tube. This degree of rarefaction was usually produced without the aid of liquid air on the charcoal. To be sure the equivalent spark length of the tube always dropped a few centimeters during the time of discharge, but a half or at most one minute of pumping was sufficient to restore the vacuum.

It may be of interest to some users of the molecular pump to know that considerable trouble was experienced with the pump due to the creeping in of oil from the bearings. Once in about six weeks the pump became stiff and the half H. P. motor was unable to drive it at the normal speed used, 8,000 R. P. M. The oil was then taken from the bearings and the whole pump thoroughly washed with filtered gasoline and dried by drawing air through it. This operation usually required three days.

The Electric Discharge.

The transformer used to produce the cathode beam was one built for the ratio 110–40,000 volts operating on a 60-cycle circuit. For a number of photographs it was used on a 440-volt 60-cycle circuit. The rays thus produced were of rather a slow velocity although the vacuum was so high that the transformer sparked across a 20 cm. gap between points outside. In order to lessen the amount of energy used and still retain the potential the transformer was operated on 110-volts D.C. with a Wehnelt interrupter. This arrangement, with or without a capacity across the interrupter, gave rays of a much higher velocity. Under these conditions, however, the equivalent spark gap of the vacuum was only about 8 to 12 cm.

The Formula.

The beam passes through uniform electrostatic and magnetic fields, whose lines are parallel to each other, and strikes the photographic plate which is lying on the lower electrostatic plate.

Let the particle be subjected to the simultaneous action of the electric and magnetic fields. The particle will be bent downward by the electric field and strike the photographic plate at a distance l (measured along the direction of the undeflected beam) from the source. It will at the same time be bent aside by the magnetic field a distance z (measured at right angles to l). Since many velocities are present they will show themselves in a long trace on the photographic plate and e/m may be calculated for any point in the trace and hence for that velocity. If the electrostatic field alone acts the resultant trace will be straight down the center of the plate. If the magnetic field also acts, then for each value of the current a trace will appear at the side and, when the current is reversed, a similar trace on the opposite side and at nearly the same distance from the center one. In photograph 58, Plate I., two values of the current were used which, with the central magnetically undeflected exposure, make five traces on the plate. The magnetic deflection, z, was taken as half the distance between two corresponding points of corresponding traces. The electrostatic plates were mounted horizontally. Each particle then describes an arc of a parabola in the vertical plane and an arc of a circle in the horizontal plane.

The Electrostatic Deflection.

Let d be the distance from the upper electrostatic plate to the upper surface of the photographic plate and let t be the thickness of the photographic plate, Fig. 3. If K is the dielectric constant of the photographic

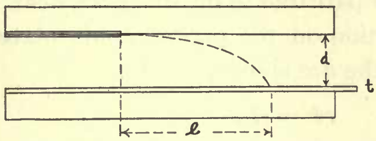

Fig. 3.

plate and V the potential difference in volts of the two electrostatic plates the force on unit charge due to the electric field is

$$E = \frac{V \times 10^8}{d + t/K}. \tag{1}$$

This force produces a downward acceleration of the electron such that

$$Ee = ma, \tag{2}$$

where e is the charge, m the mass and a the acceleration of the electron. If t_1 is the time required for the electron to fall to the photographic plate we shall have

$$vt_1 = \sqrt{l^2 + z^2}, \tag{3}$$

where v is the velocity of the electron. Equation (3) is true, since z is small enough in comparison with l that the chord may be considered equal in length to the arc. Then

$$t_1^2 = \frac{l^2 + z^2}{v^2}, \tag{4}$$

and, since

$$d = \tfrac{1}{2}at_1^2, \tag{5}$$

we get

$$d = \frac{Ee}{2m}\frac{l^2 + z^2}{v^2}. \tag{6}$$

Substituting the value of E from equation (1) and rearranging we have

$$\frac{mv^2}{e} = \frac{V(l^2 + z^2)10^8}{2d(d + t/K)}. \tag{7}$$

THE MAGNETIC DEFLECTION.

If the plane of the photographic plate be considered as in the plane of this page with the source of cathode rays at the origin, O, of the set of axes shown in Fig. 4, the arc of the circle shown will be the projection on the photographic plate of the electron's path and will show accurately the curvature of the path due to the magnetic field. Let z be the magnetic deflections, measured as previously mentioned, and let l again be the x distance to where some electron of velocity v strikes the plate. If r is the radius of curvature of the circular path due to the magnetic field, the length of the projection on the photographic plate of the actual path, or the arc shown, will be

$$r\theta = vt_1, \tag{8}$$

where θ is the angle at the center of the circle subtended by the arc.

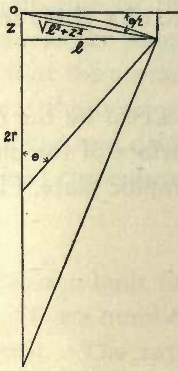

Fig. 4.

From Fig. 4 we see that

$$\tan\frac{\theta}{2} = \frac{z}{l} \tag{9}$$

and that

$$\tan\theta = \frac{l}{r - z}. \tag{10}$$

But

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \tag{11}$$

and from Fig. 4

$$\cos\theta = \frac{r-z}{r}. \tag{12}$$

Then

$$\frac{z}{l} = \sqrt{\frac{z}{2r-z}} \tag{13}$$

and

$$\frac{z^2}{l^2} = \frac{z}{2r-z}, \tag{14}$$

whence

$$r = \frac{l^2+z^2}{2z}. \tag{15}$$

The magnetic force on the particle, due to the field H, is perpendicular to the direction of motion of the particle and hence has only its component, $Hev\cos\theta$, in the y direction. Since only this component produces the deflection z, we shall find the average force, $\bar{f}$, on the particle and use this value for the magnetic force.

$$\bar{f} = \frac{Hev\int^{\theta}\cos\theta d\theta}{\theta} = Hev\frac{\sin\theta}{\theta}. \tag{16}$$

This force gives the electron an acceleration a_1 in the y direction. Then

$$\bar{f} = ma_1 = Hev\frac{\sin\theta}{\theta}, \tag{17}$$

and

$$z = \tfrac{1}{2}a_1t_1^2. \tag{18}$$

From equations (8) and (18) we have

$$t_1^2 = \frac{r^2\theta^2}{v^2}, \tag{19}$$

and from (17)

$$a_1 = \frac{e}{m}Hv\frac{\sin\theta}{\theta}. \tag{20}$$

On substitution of the values from (19) and (20) in equation (18) we get

$$z = \frac{e}{m}\frac{Hr^2\theta\sin\theta}{2v}. \tag{21}$$

From Fig. 4

$$\sin\theta = \frac{l}{r}, \tag{22}$$

and, with an approximation,

$$r\theta = vt_1 = \sqrt{l^2 + z^2}. \tag{23}$$

Substituting these values in (21) we find

$$z = \frac{e}{mv}\frac{H}{2}\sqrt{l^2 + z^2} \tag{24}$$

or

$$\frac{e}{mv} = \frac{2z}{H\sqrt{l^2 + z^2}}. \tag{25}$$

From equations (7) and (25) we obtain the desired expressions,

$$v = \frac{zV\sqrt{l^2 + z^2} \times 10^8}{Hld(d + t/K)} \tag{26}$$

and

$$\frac{e}{m} = \frac{z^2 V 2 \times 10^8}{H^2 l^2 d(d + t/K)}. \tag{27}$$

For any single photograph taken with constant deflecting fields equation (27) may be written in the form

$$z = C\sqrt{e/m}, \tag{28}$$

where C is a constant.

This equation shows the traces to be straight lines for constant values of e/m and that the outer traces should curve toward the central one for the higher velocities. From the way e/m enters the equation one would expect only slight curvature of the traces unless e/m diminished very rapidly. The equation shows that only the ratio z/l or the slope of the straight lines need be obtained from the photographic plates. This method is thus made one of particular value for the determination of e/m_0, for slow velocities, as it permits easy averaging of values.

The Earth's Field.

In fastening the apparatus to the stone pier it was carefully placed so that the undeflected beam travelled horizontally in the direction of the earth's field. The effect of the vertical component of the earth's field was then to increase the one deflection of the magnetic field and to lessen the deflection when the current was reversed. From an inspection of the equation it is seen that the effect of this vertical component may be neglected as it cancels due to the method used in measuring z.

The beam of electrons travelled from north to south so that when only the electrostatic field bends it downward it cuts the horizontal component of the earth's field at a small angle. The central trace is thus thrown a little to one side.

Let H_1 be the value of the horizontal component of the earth's field.

When the electron is deflected magnetically it has a component velocity at right angles to the magnetic field H_1 and therefore has a small force acting on it. This force will aid or oppose the force of the electrostatic field depending on the direction of the magnetic deflection. This small force due to H_1 was found to be negligible.

It follows then that a small error made in placing the apparatus such that the beam would travel neither quite horizontally nor exactly in the magnetic meridian would have no appreciable effect on the results of the experiment.

The dielectric constant, K, of the photographic plate was taken as 6. Since the plate is in contact with the lower electrostatic plate and the electrostatic field is on for ten to thirty minutes before the exposures are made the value of the constant chosen must not be that obtained by a method not allowing for the accumulation of a charge by the glass. It should be pointed out, however, that if the value of e/m_0 is measured from the same photograph the value of K will in no wise affect the value of the ratio m/m_0 if only K remain constant. It will enter, however, in the determination of the velocity of the electron but the error thus introduced is relatively small.

The deflecting magnetic field was kept at values sufficiently small that z^2 could be neglected compared with l^2. The equation for the velocity then becomes

$$v = \frac{zV \times 10^8}{Hd(d + t/K)}. \tag{29}$$

If the value of e/m is calculated from the smaller deflections, z_0 and l_0, on a photograph the ratio m/m_0 for the higher velocities is given by

$$\frac{m}{m_0} = \frac{z_0^2}{l_0^2}\frac{l^2}{z^2}. \tag{30}$$

The individual values of e/m as calculated from the photographs are shown in Fig. 5, in which (as well as in Figs. 6, 7 and 8) the full line curves marked " A " and " L " correspond to the theoretical values of Abraham and Lorentz respectively. Of the points lying above both of these curves all except three are due to a single photograph.

The ratio of the masses was also calculated by means of the preceding equation. To test which of the theoretical curves the points collectively best fit it was assumed that the value for the slowest velocity electrons showing on each of the photographs was a value exactly fitting the Lorentz curve and the other values were plotted by using only the ratio of the masses as calculated from the photographs. These values are set down in Fig. 6. Similarly Fig. 7 shows the results assuming the

value for the slowest velocity showing on each of the photographs to lie exactly on the Abraham curve. Now by a comparison of Figs. 6 and 7 it is seen that in either case the points fit the Lorentz curve more nearly than the Abraham curve.

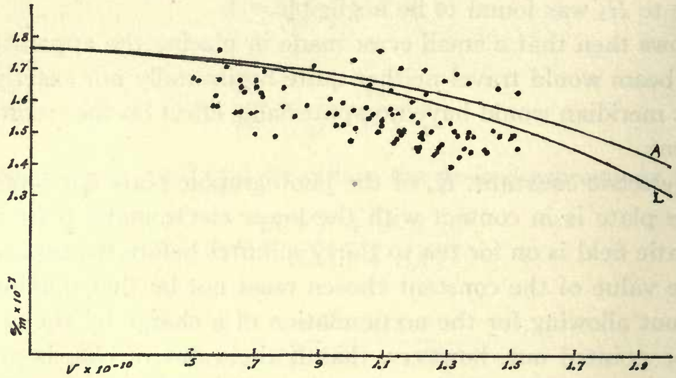

Fig. 5.

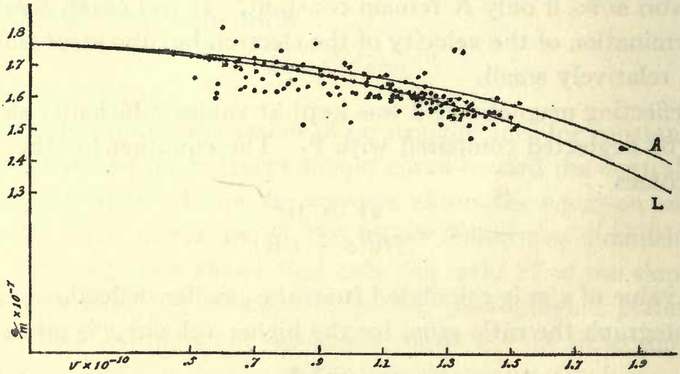

Fig. 6.

Table I. gives the data and results taken from one pair of traces on one of the photographs.

TABLE I.

l Cm.	$2z$ Cm.	z/l Cm.	$e/m \times 10^{-7}$.	$v \times 10^{-10}$.	Remarks.
3.93	.1443	.01836	1.708	.6089	
4.43	.1587	.01791	1.625	.6696	
6.93	.2503	.01806	1.653	1.056	Photograph 58,
7.43	.2667	.01795	1.632	1.125	1,926 volts,
7.93	.2817	.01796	1.599	1.189	$d = .8137$ cm.,
8.43	.2957	.01754	1.558	1.247	$d + t/K = .8418$ cm.
8.93	.3107	.01740	1.533	1.311	
9.43	.3250	.01723	1.504	1.371	
9.93	.3427	.01726	1.508	1.446	
10.43	.3583	.01718	1.495	1.512	

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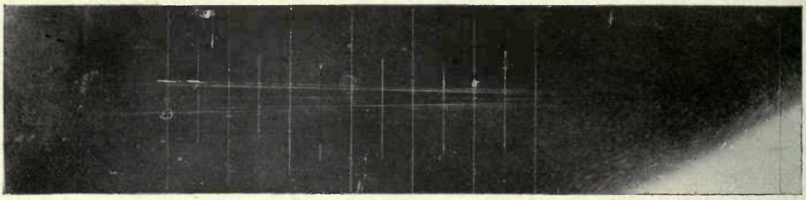
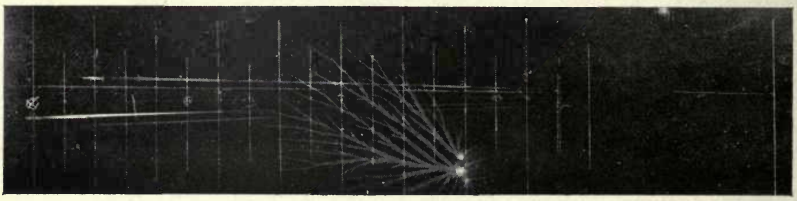
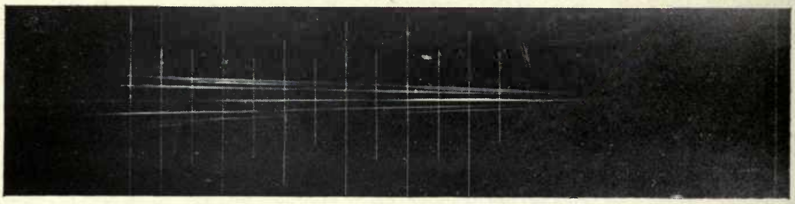


Photograph 63.



Photograph 64.



Photograph 66.

Photograph 68.

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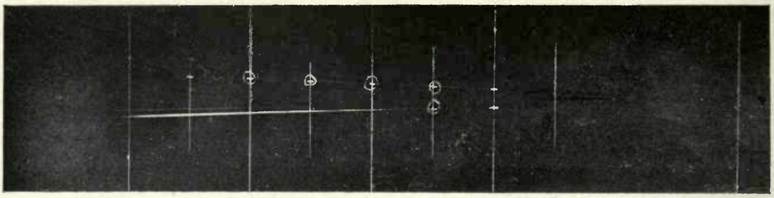
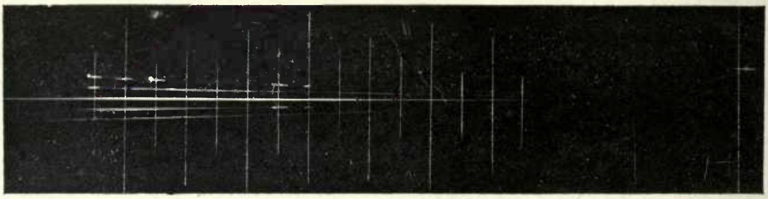
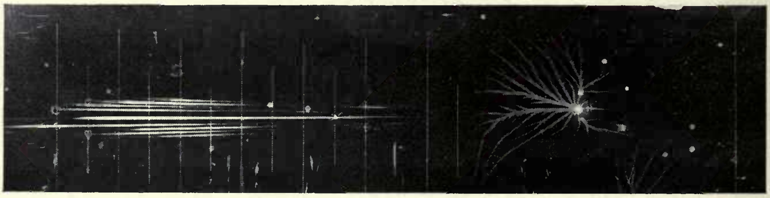


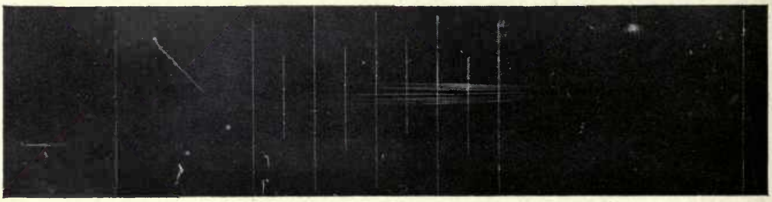
Photograph 54.



Photograph 58.



Photograph 59.



Photograph 60.

LLOYD T. JONES.

Fig. 8 represents graphically the results shown in Table I.

On each of the photographs the lines seen crossing the electron paths were drawn between the jaws of a pair of vernier calipers. The photographic plate while in position touched the ebonite disc, *L*, Fig. 1, and hence the length of the iron slip, *P*, determined the distance of the opening

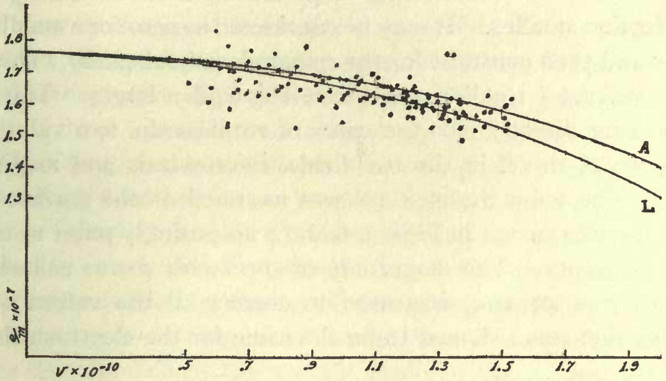


Fig. 7.

from the end of the photographic plate. On each of the photographs the line near the right is that marking the opening and the others show the successive values of *l* for which the values of *e*/*m* and *v* were calculated.

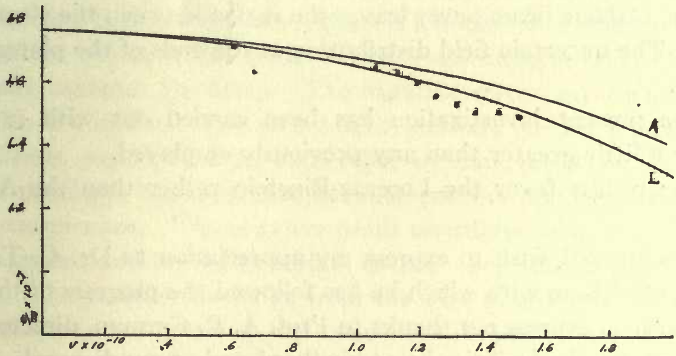


Fig. 8.

In several of the photographs, 54 for instance, the lines could be seen nicely with the unaided eye but were too dim when seen through the comparator microscope. These lines were touched with a sharp pencil and these marks used to determine the position of the lines. The photographs show very easily which of the lines were so treated.

The distance apart of the traces, $2z$, was measured by a comparator reading to .005 mm. The value of $2z$ for each distance was measured

five times, usually on different days, and the average taken. Usually no measurement differed more than .001 cm. from the average and almost never did one differ more than .002 cm. The comparator screw was calibrated.

It was found experimentally that the distortion of the magnetic field due to the iron of the constricting canal had the effect of making the magnetic deflection smaller. It may be considered as zero for a small distance p further and then constant for the remainder of the path. This has the effect of making l smaller and hence e/m and v larger. The length l does not enter directly into the value of v unless the two values of l for the distance of travel in the two fields, electrostatic and magnetic, are different. The value 1.765×10^7 was assumed as the correct value of e/m_0 and the two curves in Figs. 5, 6 and 7 accordingly point to this value for slow velocities. The magnitude of the factor p was calculated and this value, $p = .07$ cm., was used to correct all the values of l used. This correction was assumed to be the same for the electrostatic field.

Conclusions.

1. The method used in the present investigation does not necessitate a homogeneous cathode beam.

2. Each photograph gives a trace of all velocities present and makes possible a verification of the law from a single photograph.

3. The cathode beam never leaves the region between the electrostatic plates. The uncertain field distribution at the ends of the plates is thus avoided.

4. The present investigation has been carried out with rays of a velocity ɐ little greater than any previously employed.

5. The results favor the Lorentz-Einstein rather than the Abraham formula.

In conclusion I wish to express my appreciation to Dr. C. T. Knipp for the enthusiasm with which he has followed the progress of the work. Also I wish to express my thanks to Prof. A. P. Carman, director of the laboratory, for the facilities he so kindly placed at my disposal.

Laboratory of Physics,
University of Illinois,
March 1, 1916.

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ON THE INITIAL CONDITION OF THE CORONA DISCHARGE.

BY JAKOB KUNZ.

THE glow discharge of electricity surrounding the transmission wires under the influence of high potential differences has been studied by electrical engineers of England and America in recent years. In the majority of these investigations alternating current has been used, and very definite empirical results have been obtained. Comparatively few researches have been carried out on the direct corona, among which those of Watson, Schaffers and S. P. Farwell[1] may be mentioned. Farwell especially has shown that the phenomena are far more complicated than what has been revealed by the use of alternating potentials.

Before making an attempt at an explanation of some of the phenomena I wish to describe briefly the various forms of the corona due to direct current potentials. There is hardly another electric phenomenon which shows the difference between positive and negative electricity in so many different ways as the corona. There are electric, optical and mechanical differences. For very small wires the negative glow appears before the positive, for larger sizes the positive glow appears before the negative. The boundary between the two regions is a diameter of about 0.075 mm. The positive corona in air forms a very even uniform layer of light of practically constant thickness. The negative corona on the other hand starts also in a uniform layer of red light, but very quickly breaks up into bright beads, separated from each other by dark intervals. Especially at lower pressures this difference between positive and negative polarity is very conspicuous. The negative beads distribute themselves in nearly equal intervals and are fairly stable, so that they can be photographed readily. The positive discharge from the wire in a coaxial cylinder has never been found to break up into beads, but if the discharge takes place between two parallel wires, then at higher potentials the positive column of light also breaks up into shorter intervals and finally into beads. The question arises as to whether the beads are connected with irregularities on the surface of the wire, or whether it is an intrinsic phenomenon, independent of the surface irregularities. The number of beads depends on

[1] Watson, Electrician, London, Vol. 63, p. 828, 1909; Vol. 64, p. 707 and 776, 1909–10.
Schaffers, Comptes rendus, July, 1913, p. 203.
S. P. Farwell, Transactions of American Institute of Electrical Engineers, Nov. 13, 1914.

the pressure of the gas and on the potential difference. If these two variables are kept constant, the number of beads remains constant, indicating that the beads on smooth wire are an intrinsic phenomenon of the negative discharge. In addition special experiments have been performed with polished and chemically corroded silver wires in order to test this conclusion. For a given potential difference the corona current increases with decreasing diameter of the wire, and with decreasing pressure.

The characteristic curve, like the starting point of the corona, depends on the polarity and diameter of the wire. For wire smaller than 0.077 mm. diameter the current from the negative wire is greater than that from the positive wire. For the diameter of 0.077 mm. the currents for opposite polarity coincide accurately over a certain range of voltages above the critical voltage, and then the negative current becomes and remains the larger. For sizes of wire larger than 0.077 mm. the curves for the two signs cross each other. For the lower potential differences the positive current is the greater, and for higher potential differences the negative current is the greater. Previous investigators already found that the relation between the electric force E at the surface of the wire and the radius R_1 is given by the formula

$$E = E_0 + \frac{b}{\sqrt{R_1}},$$

where E_0 and b are constants, E is calculated by the electrostatic formula:

$$E = \frac{\Delta V}{R_1 \log \frac{R_2}{R_1}}$$

ΔV being the potential difference between the central wire and the cylinder of radius R_2. For the smallest sizes of wires used, the relation between E and R_1 ceases to hold as can be seen from Table I.

The critical voltage required to produce visible corona depends not only on the radius of the wire, but also on the pressure. At very low pressures the negative corona starts before the positive one, at higher pressures the positive corona starts at first. The characteristic curves also depend on the pressure. The pressure of the gas and the radius of the wire play an analogous rôle, very thin wires seem to correspond to low pressures. The relation between the pressure p of air, the radius R_1 of the central wire and the critical electric force E at the surface of the wire is given as follows:

$$E = p\left(E_0 + \frac{b}{\sqrt{pR_1}}\right),$$

where E_0 and b are constants. This relation holds as far down as 53 mm. Hg pressure for the positive corona: the constants E_0 and b have different values for the positive and negative wire.

Table I.

R cm.	$V+$Volts.	$E+$Volts per cm.	$E+$Calcul.	$V-$Volts.	$E-$Volts per cm.	$E-$Calcul.
0.00135	2,720	2.74×10^5	2.62	2,520	2.52×10^5	2.55
0.00218	3,380	2.58	2.29	3,230	2.45	2.23
0.0023	3,500	2.25	2.09	3,300	2.08	2.04
0.00258	3,630	2.12	1.99	3,500	2.02	1.94
0.00386	4,060	1.66	1.67	4,060	1.66	1.65
0.00678	5,140	1.31	1.34	5,320	1.36	1.33
0.00825	5,710	1.25	1.25	6,140	1.21	1.21
0.012	6,600	1.07	1.09	6,840	1.09	1.09
0.013	7,180	1.07	1.06	7,660	1.14	1.06
0.0205	8,900	0.93	0.91	9,370	0.99	0.92
0.0325	10,880	0.80	0.79	11,440	0.83	0.80
0.0385	11,850	0.77	0.75	12,400	0.79	0.76
0.0512	13,500	0.71	0.69	14,120	0.73	0.71
0.0642	14,700	0.65	0.65	15,220	0.64	0.64

When the corona starts, the pressure of the gas increases suddenly. We shall call this pressure ionization pressure. It can easily be measured by means of a sensitive U-tube open manometer. This increase of the pressure is very distinctly different from the increase of the pressure due to the evolution of Joule's heat. As soon as the current is interrupted the ionization pressure sinks suddenly down to zero, while the other pressure increases and dies out gradually. The ionization pressure is in general for a given potential difference larger when the wire is negative than when it is positive, but the difference is very small, if not opposite at the beginning of the corona. The ionization pressure is very nearly proportional to the current, especially when the wire is positive.

The ionization pressure as well as the fact that a higher potential difference is necessary to start the corona for thicker wires can be used with advantage for the construction of voltmeters, some of which are in use in the laboratory of the University of Illinois.

It has been mentioned that the negative electricity leaves the wire in the form of very beautiful beads or brushes, mostly evenly spaced along the wire. The number of brushes per unit length depends on the pressure and on the potential difference. With increasing pressure and with increasing potential difference the number of beads per unit length increases and their brightness at the same time decreases. The beads start from a point of the wire and spread out fanlike in a plane at right

angles to the wire. Very interesting is the influence of a short arc in series with the tube upon the character of the positive and negative discharges. The very well defined positive layer of light spreads out considerably and the negative brushes disappear almost entirely, giving room to a continuous glow, whose boundary is ill defined; in other words, a very short spark in series with the discharge tube destroys the difference in the appearance between the positive and the negative corona. This is due to the superposition of a high frequency alternating or intermittent current. A small change of the spark length between the corona and the dynamos produces very marked differences in the luminous discharge. For a certain spark length the corona assumed the form of bright streamers which fill the entire space between the wire and the cylinder. If the spark length is slightly changed, these streamers concentrate into a few luminous bands, about equally spaced, whirling round the wire and presenting a very beautiful aspect. If the potential difference is slightly increased, this phenomenon is replaced by the arc, which is apparently the more stable form of discharge. With the introduction of a spark a hissing sound will be heard from the corona tube.

The difference between positive and negative elctricity makes itself felt finally in mechanical effects. When the corona takes place between two parallel wires which are not stretched too strongly, the negative wire bows in toward the positive and the positive bows away from the negative. When the wires are purposely made rather slack the positive wire vibrates strongly with a circular motion, while the negative wire remains motionless.

The field between two parallel wires and between a cylinder and a coaxial wire has been explored by means of a third platinum electrode. Even before the corona started there was found a distortion of the electrostatic field especially in the neighborhood of the electrodes; and in many if not in all cases the electric force at the surface of the wires is different from the calculated value in the moment when the corona arises. The observed electric force seems to be larger than that calculated from the electrostatic formula. When the field is studied in the space between the central wire and the coaxial cylinder, it becomes very difficult to explore the neighborhood of the negative wire, where the potential seems to be subject to continuous changes. The exploration of the field around the positive wire offers no difficulties.

The explanation of the large variety of phenomena described is far from being complete. An attempt at an explanation of some of the phenomena will be made. One might expect that luminous discharge begins when the electric force or polarization on the surface of the wire

obtains a constant value required for the ionization of the molecules. It has been found however that the critical electric force is given by the expression:

$$E_1 = E_0 + \frac{b}{\sqrt{R_1}}.$$

Various values for E_0 and b have been given, for instance, $E_0 = 30$, $b = 9$ by Y. S. Townsend. Farwell found that the values of E_0 and b are distinctly different for positive and negative wires. For positive wires he found $E_0 = 31.6$; $b = 8.43$. For negative wires $E_0 = 35.0$; $b = 8.06$. We shall now assume that in the neighborhood of the wire in a layer of constant thickness δ a certain constant energy is required for the beginning corona, different for positive and negative electricity; indeed the splitting up of the molecules into ions and the emission of light requires energy. When a sufficient amount of energy is supplied, the luminous discharge called corona will occur. It has been shown by Schaffers that the thickness $\delta = 0.07$ cm. of the luminous layer is independent of the radius of the wire. In the neighborhood of the wire, the electric force E assumes large values so that the polarization also is large and an opposing electric force E_0 will be created, so that the resultant electric force is equal to $E - E_0$. If k is the dielectric constant, R_1 the radius of the wire, then we have:

$$E_g = \frac{k}{8\pi} 2\pi R_1 \delta (E - E_0)^2,$$

$$E = E_0 \sqrt{\frac{4E_o}{kR_1\delta}}.$$

If E_g, k and δ remain constant, then we have

$$E = E_0 + \sqrt{\frac{4E_g}{k}} \frac{1}{\sqrt{R_1}},$$

the rule established by the engineers. E_0, E_g and δ are obviously different for the two polarities of the wire. In favor of this theory is the phenomenon of beads. When a thin film of liquid is formed along a thin thread, the film on account of the surface tension breaks up into beads; similarly when a layer of electric energy is formed on the surface of the wire, it will have the tendency of breaking up into beads extending further away from the wire than the original layer. The fact that the negative discharge is much more apt to form beads than the positive one, seems to be connected with the mechanism of the discharge itself. When the wire is very thin negative electricity escapes easier than the positive one, just as in the case of very sharp points and at very low pressures.

The negative electricity seems to escape both from the molecules of the gas and of the metal, while the positive electricity consists only of positive ions, formed in the air alone, as no positive ions escape from the metal. The positive current consists of positive ions alone, the negative current of negative ions and electrons. Now it seems easier for the electrons to escape in a few places from the metal in large quantities, than from the entire surface of the wire in smaller quantities. That electrons escape from the neighborhood of the negative wire is also indicated by the fact that the negative wire bows in toward the positive one, which bows away from the negative one and that under the same circumstances the negative wire remains almost motionless while the same wire, when charged positively, carries out rotations of large amplitude.

For very small wires as well as for low pressures the negative corona starts before the positive one; for larger wires and higher pressures the positive corona starts before the negative. The negative electricity seems to escape in the form of electrons easier from thin metal wires than from molecules of the air. This phenomenon suggests that the average mass of the ions from small negative wires is smaller and the mobility larger than from larger negative wires.

Y. S. Townsend[2] has given another theory of the initial conditions of the glow discharge from wires, where he assumes the same values of the constants E_0 and b; and applies the same theory to the corona as to the spark discharge. The two phenomena are however in many respects different.

The law for ionization of a gas by collision can be expressed as follows:

$$\frac{\alpha}{p} = f\left(\frac{E}{p}\right).$$

Y. S. Townsend made an interesting application of this rule, based on experiments at low pressures, to the corona and spark discharge, which phenomena he considers as due entirely to the same process of ionization. Let us choose the following relations between the two cylinders in which

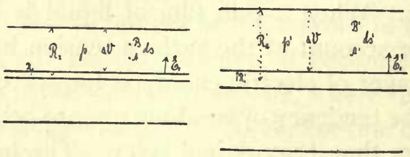

Fig. 1.

the corona occurs,

$$R_1' = zR_1,$$
$$R_2' = zR_2,$$

[1] Y. S. Townsend, The Electrician, June 6, 1913, p. 348.

$$p' = \frac{p}{z},$$

$$ds = zds',$$

z being a given constant number. $V = \Delta V$ is equal to the potential difference applied in both cases:

$$E_1 = \frac{V}{R_1 \log \frac{R_1}{R_1}}; \quad E_1' = \frac{V}{R_1' \log \frac{R_1'}{R_1'}} = \frac{V}{zR_1 \log \frac{R_1}{R_1}} = \frac{E_1}{z}.$$

$E_2 = E_1/z$ holds not only on the surface of the inner wire but in any two corresponding points such as A and A' or B and B'.

The number of ions formed by collision when a negative ion travels over the distance $AB = ds$ is given by:

$$\alpha ds = pdsf\left(\frac{E_1}{p}\right).$$

The number of ions produced in the second experiment over the distance ds' is given by:

$$\alpha' ds' = p'ds'f\left(\frac{E_1'}{p'}\right) = \frac{p}{z} zdsf\frac{\frac{E}{z}}{\frac{p}{z}},$$

$$\alpha' ds' = pdsf\left(\frac{E_1}{p}\right) = \alpha ds,$$

a negative ion traveling through the distance ds produces the same effect as over the distance ds'. The same holds for the collision of positive ions.

$$\beta ds = \beta' ds',$$

hence we have the same effects in both tubes. A given potential difference V causes the same phenomena in both tubes. If V is sufficient to start corona in one cylinder, it will also give rise to it in the other cylinder. If

$$R_2'p' = zR_2\frac{p}{z} = R_2 p;$$

if

$$R_1'p' = R_1 p$$

and if

$$V = V$$

then

$$R_1'E_1' = E_1 R_1.$$

$R_1 p$ is therefore only a function of $R_1 E_1$. If we keep $R_1 E_1$ constant, $R_1 p$ remains constant.

This theory applies to the beginning spark as well as to the beginning glow discharge. It does not give an answer to one of the first questions regarding the corona discharge, namely, is the current due to ionization by negative or positive, or to both ions?

Now the following relation holds between the critical electric force E_1 and the radius R_1

$$E_1 = E_0 + \frac{b}{\sqrt{R_1}},$$

$$E_1 R_1 = E_0 R_1 + \frac{bR_1}{\sqrt{R_1}}, \qquad \text{for } p = 1,$$

or

$$E_1 R_1 = E_0 R_1 \cdot 1 \cdot + \frac{bR_1 \cdot 1 \cdot}{\sqrt{R_1 1 \cdot}}.$$

But if we keep $R_1 \cdot 1 \cdot = R_1 p =$ constant, then $E_1 R_1$ remains constant.

$$E_1 R_1 = E_0 R_1 p + \frac{bR_1 p}{\sqrt{R_1 p}},$$

$$E_1 = E_0 p + \frac{bp}{\sqrt{R_1 p}},$$

$$E_1 = p\left(E_0 + \frac{b}{\sqrt{R_1 p}}\right),$$

a rule that has been established by the engineers, before the theory was developed.

TABLE II.

p in Mm.	$+V$ in Volts.	$+E_1$ Volts per Cm. $\times 10^4$.	$+E_1$ Calcul.	$-V$	$-E_1$	$-E_1$ Calcul.
2.	720	0.765	0.33	580	0.615	0.33
10.9	940	0.998	0.80	870	0.925	0.81
18.9	1,110	1.18	1.07	1,200	1.275	1.08
53.2	1,770	1.88	1.88	1,920	2.04	1.94
91.3	2,350	2.50	2.56	2,580	2.74	2.64
173.5	3,450	3.60	3.72	3,750	3.99	3.86
248.5	4,250	4.51	4.65	4,610	4.90	4.84
334.8	5,120	5.42	5.58	5,520	5.86	5.86
483.6	6,660	7.08	7.11	7,120	7.55	7.45
616.6	7,800	8.29	8.33	8,330	8.85	8.77
720.0	8,730	9.28	9.21	9,210	9.80	9.72
746.0	8,980	9.51	9.51	9,530	10.1	10.1
768.3	9,100	9.67	9.65	9,640	10.2	10.2

Table II. gives as function of the pressure the electric force at the surface of the wire given by the potential difference V and the radii and cal-

culated by the last formula for the positive and negative wire. For the positive wire the rule holds from atmospheric pressure down to 53 mm. or lower, while for the negative wire the deviations are noticeable for much higher pressures. For the positive wire the constants $E_0 = 31.6$; $b = 8.43$ and for the negative wire $E_0 = 35.0$; $b = 8.06$ have been used, values that have been determined in a previous set of experiments.

If ionization by collision were due to negative ions alone, the constants E_0 and b would be the same; their difference indicates, that either negative and positive ions act as agents of ionization by collision or that another source of ionization is involved in the beginning corona.

It has been mentioned that the pressure of the gas suddenly increases, when the corona appears. We shall apply the principle of conservation of energy to this phenomenon as follows: Let the pressure of the gas in the corona tube be equal to p_0, let the potential difference applied be equal to ΔV, the current i and the volume v; the pressure will rise from p_0 to p_1; the work done by the current is equal to ΔVi and consists in the ionization of the gas plus the radiation of light U, and a reversible part W_1

$$\Delta Vi = U + W_1.$$

It is evident that if the current increases the current must decrease with increasing pressure, otherwise a finite amount of electric work $\Delta V \cdot i$ could perform an infinite amount of mechanical work.

After this first process we shall increase the pressure from the outside by dp_1 by means of the work

$$dW_2 = vdp_1,$$

so that the pressure rises from p_1 to $p_1 + dp_1$. Now we shall try to reach the same final state of the gas starting from the same initial conditions. Only this time we shall apply external pressure at first, supplying the work:

$$dW_3 = vdp_0,$$

while the pressure rises from p_0 to $p_0 + dp_0$. Now the same potential difference ΔV applied will produce a current i', and the work done by the current will be equal to:

$$\Delta Vi' = U' + W_4$$

and the pressure will rise to $p_1 + dp_1$. If the initial and the final conditions are the same, then the principle of conservation of energy leads to the following equation:

$$\Delta Vi - dW_2 = \Delta Vi' - dW_3,$$

$$U + W_1 - vdp_1 = U' + W_4 - vdp_0.$$

$$\Delta V(i - i') = v(dp_1 - dp_0).$$

But

$$i' = i - di,$$

$$\Delta V di = vd(p_1 - p_0)$$

$$i = \frac{v}{\Delta V}(p_1 - p_0) + i_0,$$

that is, the current will decrease with increasing pressure p_1, and hence the ionization pressure p_1 will increase proportionally to the current. Experiments will soon be published which will show that this conclusion holds in a wide range of currents and ionization pressures.

SUMMARY.

A description of the numerous phenomena has been given which are connected with direct current corona. The difference between positive and negative electricity appears in electrical, optical, mechanical and probably chemical effects. A new attempt at an explanation of some of the relations disclosed by experiments has been made. Relations between the critical electric force at the surface of the wire, the pressure of the air, and the radius of the wire have been obtained. The constants in these relations are different for the positive and the negative wire. The principle of conservation of energy has been applied to the ionization pressure and it is predicted that over a certain range the current should be directly proportional to the pressure increase.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
March, 1916.

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DETERMINATION OF THE LAWS RELATING IONIZATION PRESSURE TO THE CURRENT IN THE CORONA OF CONSTANT POTENTIALS.

BY EARLE H. WARNER.

INTRODUCTION.

THE "corona" is the glow which surrounds conductors when there exist high potential differences between them and neighboring bodies. A careful study of the corona phenomena is necessary (1) to determine the factors which regulate the loss of power due to the corona, which on long transmission lines may be an important item, and (2) to obtain data from which a theory can be developed which will, with mathematical rigor, explain the corona effects. The first of these objects has been quite successfully carried out by Peek, Whitehead, Ryan and others. The only advances toward a theoretical explanation of the corona have been made by Bergen Davis[1] and Townsend.[2] In these two theories the authors have assumed that the corona is an ionization phenomenon. That is, they assume that the high potential difference causes the few ions which are always present in a gas to move with a velocity sufficiently great to break the molecules with which they collide into two parts, one bearing a positive charge and one a negative charge. All these charged particles then move, because of the influence of the field, toward one or the other of the terminals. The presence of these ions thus explains the conductivity of the gas and the acceleration of the ions explains the light effect. If the corona is an ionization phenomenon one would expect, if the corona apparatus was inclosed, at the instant the corona appeared, *i. e.*, at the instant the molecules were broken up into ions, that the pressure in the apparatus would increase; because according to kinetic theory the greater the number of particles in a given volume the greater the pressure. This pressure increase was first discovered by Dr. S. P. Farwell,[3] working in this laboratory. The above mentioned theories assume ionization but do not account for such a pressure increase. Under certain circumstances this pressure increase can amount to as

[1] "Theory of the Corona," Proc. A. I. E. E., January, 1911.

[2] "The Discharge of Electricity from Cylinders and Points," Phil. Mag., May, 1914.

[3] "The Corona Produced by Continuous Potentials," Proc. A. I. E. E., November, 1914.

much as three cm. of mercury. This pressure increase can not be due to the heating effect of the current, because it occurs very quickly and if the current is broken in a few seconds, the pressure at once returns to its initial value. The heating effect of the current becomes noticeable only after several seconds and then when the current is broken the pressure does not at once return to its initial value but it requires some time for the heated gas to cool off. Since the conception of ionization is so intimately associated with the idea of increase in pressure, it seemed important to determine the laws relating this ionization pressure to the corona current.

Theory.

Dr. J. Kunz has developed a theory which predicts how this pressure increase should vary with the current. One can best understand his development by thinking of the corona as occurring around a wire which is coaxial with a cylinder. See Fig. 1, which represents a cross section

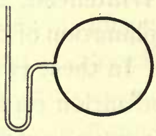

Fig. 1.

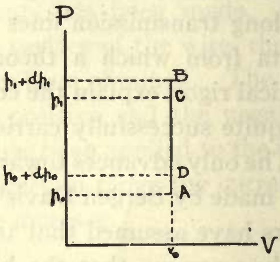

Fig. 2.

of such a corona tube. Suppose the ends of the tube to be closed, so as to inclose a constant volume v_0. When the wire is connected to a very high positive potential and the case grounded the corona glow appears around the wire and the pressure instantly increases from atmospheric to some higher value. Let the condition of the gas at the beginning of the experiment be represented by the point A, on the $p-v$ plane. (See Fig. 2.) The volume is then v_0 and the pressure p_0.

Step I.—Apply a potential difference e between the wire and the case. Some current i will flow and the pressure will immediately jump from p_0 to a higher value, say p_1. The state of the gas will now be represented by the point C. The work done by the current per second, ei, will then be equal to the increase of internal energy of the gas ΔU, plus the work done by the gas W_1, due to the pressure increase. This energy equation gives us

$$ei = \Delta U + W_1. \tag{1}$$

Step II.—Let us force into the tube a small amount of gas. This

will require work dW_2 and the pressure will increase from p_1 to $p_1 + dp_1$ and can be represented by the state point B. Then

$$dW_2 = - v_0 dp_1. \tag{2}$$

The total work to change the gas from state A to state B has then been

$$ei + dW_2 = \Delta U + W_1 - v_0 dp_1. \tag{3}$$

Now let us start again with the same initial conditions and by two different steps arrive at the same final condition.

Step III.—When the state of the gas is A let us force in a small amount of gas. This will require work dW_3 and the pressure will increase from p_0 to $p_0 + dp_0$, which may be represented by the state point D. Then

$$dW_3 = - v_0 dp_0. \tag{4}$$

In the existing conditions the size of the current depends not only on the potential difference but also upon the initial and final pressures. The increase in current causes an increase in pressure which tends to stop the current. The steady condition of the current represents a condition of equilibrium between the attempt of the current to increase the pressure and the attempt of the increased pressure to stop the current.

Step IV. Now apply the same potential difference e. Let that current i' flow so that it will cause the pressure to increase from p_0+dp_0 to $p_1 + dp_1$, that is, so that the state of the gas can be represented by B. Then as in Step I.

$$ei' = \Delta U' + W_4. \tag{5}$$

In the last two steps the total work required to change the state of the gas from A to B is

$$ei' + dW_3 = \Delta U' + W_4 - v_0 dp_0. \tag{6}$$

Then by the law of the conservation of energy, the work required to change a system from one state to another is independent of the path, we have

$$\Delta U + W_1 - v_0 dp_1 = \Delta U' + W_4 - v_0 dp_0 \tag{7}$$

or

$$\Delta U - \Delta U' + W_1 - W_4 = v_0(dp_1 - dp_0). \tag{8}$$

Subtracting (5) from (1) we have

$$\Delta U - \Delta U' + W_1 - W_4 = e(i - i'). \tag{9}$$

Therefore

$$e(i - i') = v_0(dp_1 - dp_0). \tag{10}$$

But

$$i = i' + di.$$

Then

$$edi = v_0 d(p_1 - p_0) \tag{11}$$

and integrating

$$i = \frac{v_0}{e}(p_1 - p_0) + \text{a constant.} \tag{12}$$

Since $(p_1 - p_0)$ represents the increase in pressure, that is, the ionization pressure, this equation shows that the ionization pressure should be exactly proportional to the corona current.

It was the object of the experiments which have been performed to test this relationship with pure gases in the tube.

Apparatus.

The constant potentials were obtained from a battery of continuous current shunt-wound 500-volt generators connected in series.

The corona tube was of the wire and coaxial cylinder type. (See Fig.

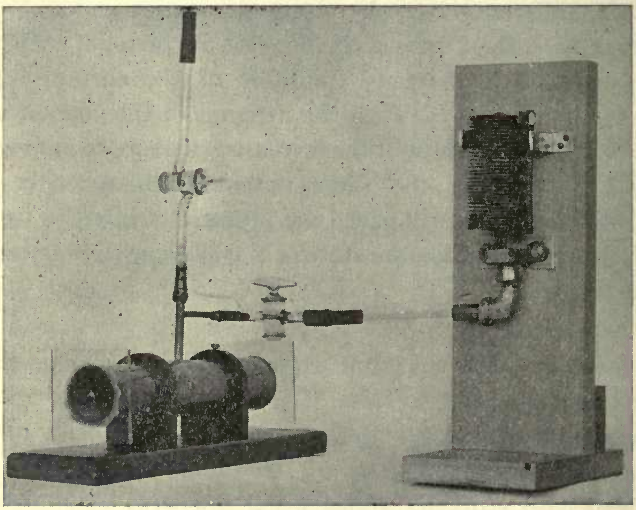

Fig. 3.

3.) Glass plates with holes for the wire to pass through were sealed to the ends of the tube so that the holes were on the axis of the cylinder. The wire, No. 32, copper, passed through the holes and was thus coincident with the axis of the cylinder. The wire was sealed into these holes and held taut by red sealing wax. To the cylinder was soldered a small " T " tube, one side of which was joined to the vacuum pump and the other side was connected to a Bristol aneroid pressure gauge.

The increase in pressure was measured by this Bristol gauge. Any increase in pressure caused it to bend slightly and so rotate the mirror. By observing the deflection of a beam of light over a scale, which had

previously been calibrated by reading simultaneously the deflected beam and a water manometer connected directly to the gauge, the increase in pressure in cm. of water could be determined. The advantage of such a pressure measuring instrument in this experiment is that it is very quick in its action. The instant the pressure increases the gauge jumps right up to its new position and a reading can be taken in a very few seconds. It was necessary to read this pressure increase quickly because if much time was required, the heating effect of the current would increase the pressure also.

The current was measured by a Type *H* D'Arsonval galvanometer. The apparatus was connected as is shown in Fig. 4.

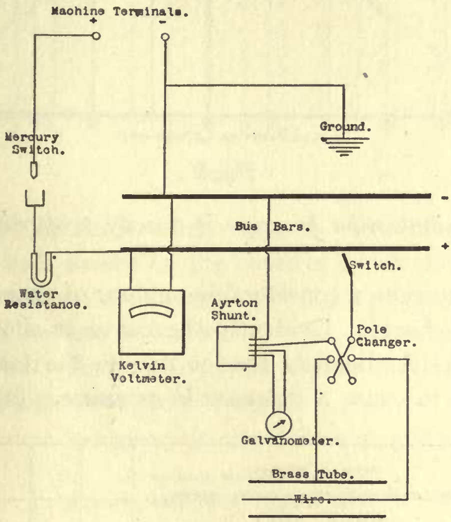

Fig. 4.

Discussion.

Experiments were made when the wire was positive and the case grounded with dry air, hydrogen, nitrogen, carbon dioxide, oxygen and ammonia as the gases in the tube. Considerable care was taken to see that these gases were absolutely pure. They were all dried carefully before they were used. The following curves (Figs. 5, 6, 7, 8, 9) show graphically the results. Fig. 10 shows all the curves plotted to the same scale. With this scale the hydrogen curve should be continued until its ordinate is equal to that of the carbon dioxide curve.

The fact that the points all lie so accurately on a straight line shows conclusively that experiment verifies the prediction made by Dr. Kunz's theory. The law can then be stated that, in the gases studied with the

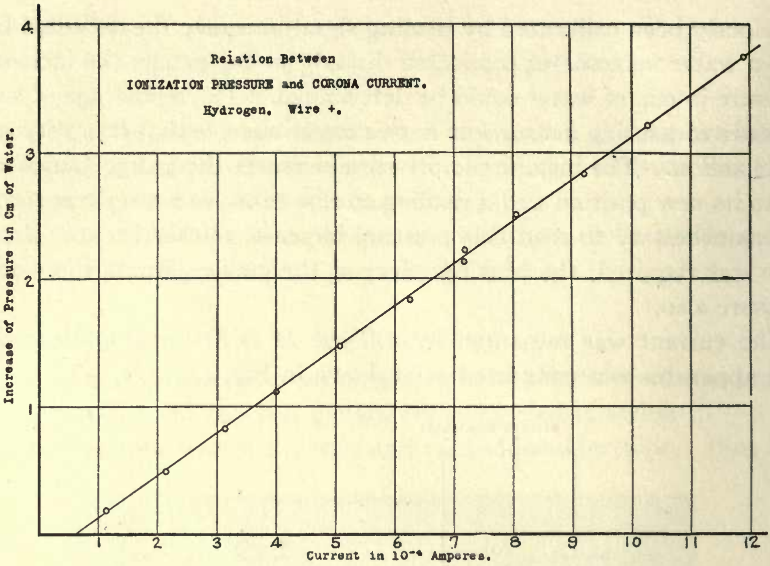

Fig. 5.

wire positive *the ionization pressure is exactly proportional to the corona current.*

In the case of oxygen a considerable amount of ozone was formed due to the corona discharge. Evidently the curve as shown is a resultant of two effects: (1) A chemical change due to the formation of ozone. This would tend to cause a decrease in pressure. (2) The increase in

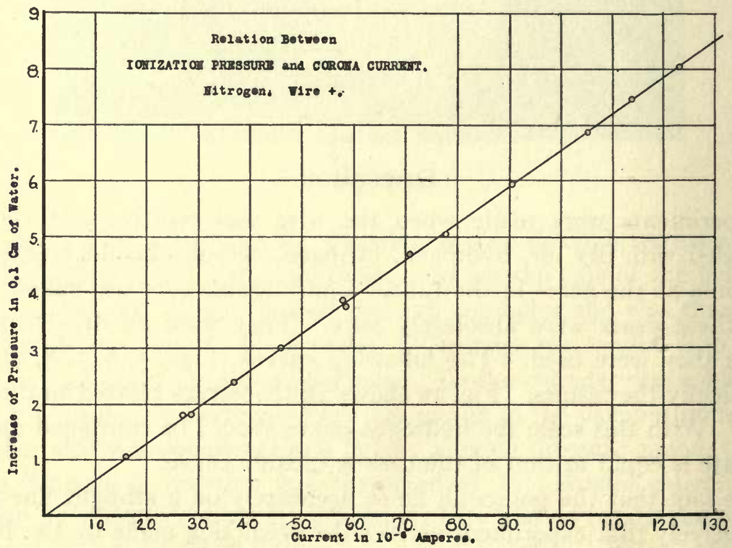

Fig. 6.

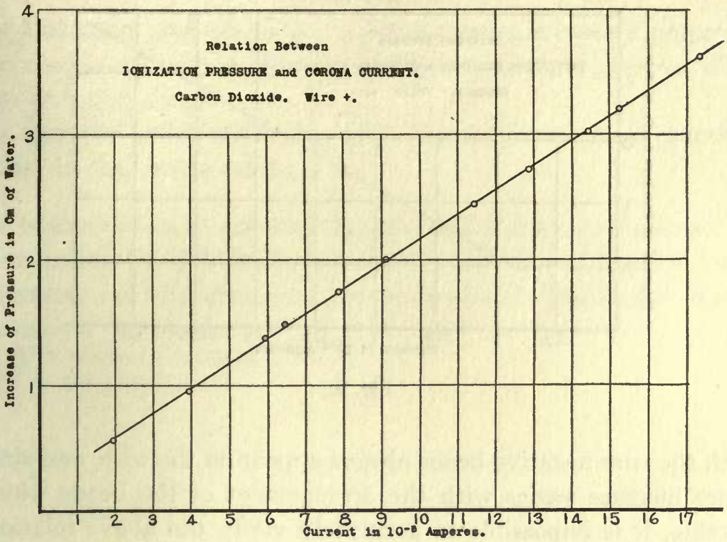

Fig. 7.

pressure due to the ionization of the oxygen. Since the ionization curve is a straight line, as is shown by the gases in which probably there is no chemical action, and since this resultant curve of oxygen is a straight line, the following law can be stated:

Whenever chemical change takes place due to the corona the chemical change is exactly proportional to the current.

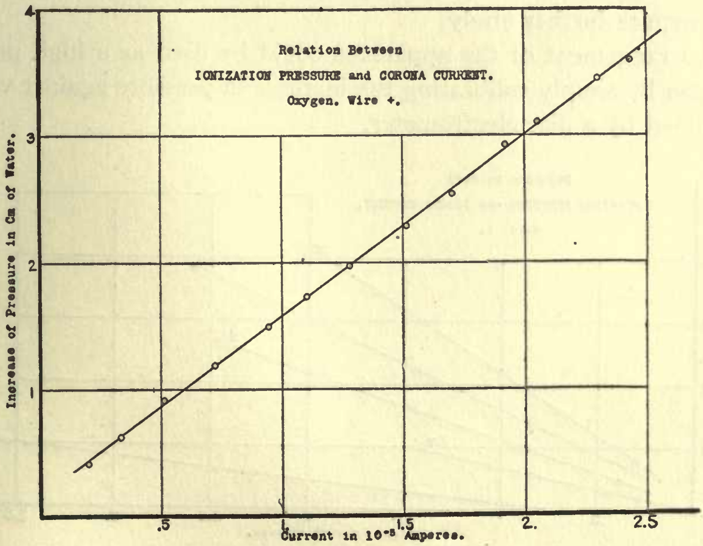

Fig. 8.

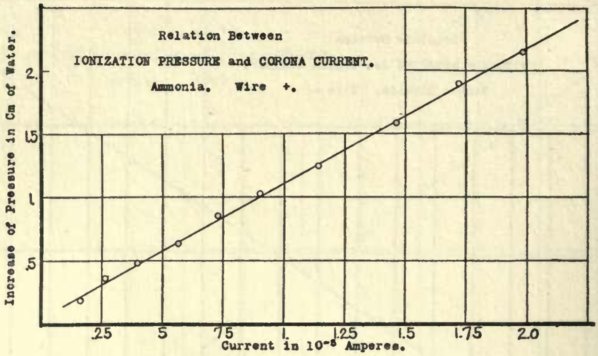

Fig. 9.

With the wire negative beads always appear on the wire, and since the pressure increase varies with the arrangement of the beads which are not stable, it is impossible to accurately verify the above relationship. When, instead of the quick acting gauge, an ordinary open manometer which is slow in its action was used, it was discovered that the same relationship as above stated is very nearly true for the wire negative as well as positive.

The increase in pressure in the case of nitrogen, showing ionization, is one of the exceptional cases where nitrogen is largely ionized at low temperatures and thus probably chemically active.

How nitrogen, carbon dioxide and ammonia are ionized, are questions which require further study.

The arrangement of the apparatus could be used as a high potential voltmeter by simply calibrating the increase in pressure against volts, as determined by a disc electrometer.

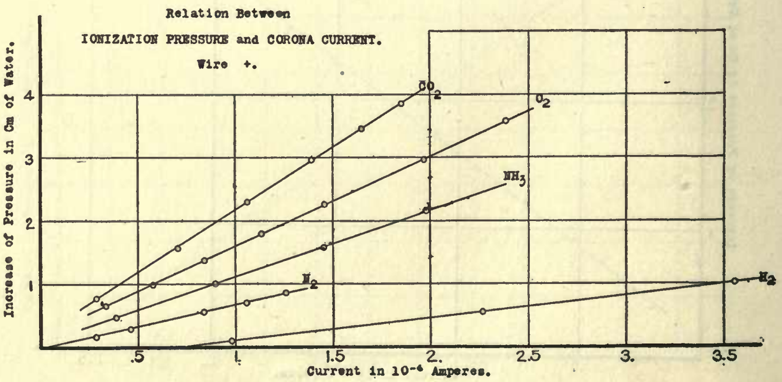

Fig. 10.

Summary.

The ionization pressure in the positive corona is exactly proportional to the corona current in dry air, hydrogen, nitrogen, carbon dioxide, oxygen and ammonia.

Any chemical action that takes place due to the corona is exactly proportional to the corona current.

The writer wishes to acknowledge his indebtedness to Professor A. P. Carman and to Dr. Jakob Kunz, associate professor of physics, for their deep interest and helpful suggestions concerning the conduct of this work.

Laboratory of Physics,
University of Illinois,
May, 1916.

DIRECT CURRENT CORONA FROM DIFFERENT SURFACES AND METALS.

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DIRECT CURRENT CORONA FROM DIFFERENT SURFACES AND METALS.

BY SYLVAN J. CROOKER.

I. INTRODUCTION.

IT has been shown by F. W. Peek[1] and by S. P. Farwell[2] that the corona discharge is quite different when the wire is positive and when it is negative. The starting point of the corona as well as the characteristic curves depend on the polarity of the wire. When the wire is negative the corona assumes the form of bright " beads " which are strung along the wire more or less evenly, the number of the beads per unit length depending on the pressure of the gas and the potential difference between the wire and the cylinder. This beautiful but complicated phenomenon suggested that probably the surface conditions and the chemical nature of the wire might influence at least the negative corona in the form of beads.

It became the purpose of these experiments then to find out the influence of the surface condition of the wire upon the starting point and the characteristics of the corona discharge phenomena.

The apparatus used consisted of a metal cylinder (inside diameter 3.63 cm.), with a longitudinal slot for observation (1.53 cm. wide), sealed in a glass cylinder and arranged in such a manner that wires of different sizes could be easily strung along the cylinder axis. It was possible to readily connect the tube to a vacuum pump for varying the pressure. The high potential direct current was taken from forty 500-volt D.C. generators connected in series. The machines were self-exciting and could be cut in or out by closing or opening the field switches. Smaller variations than 500 volts could be obtained by varying the speed of the driving motors or by the adjustment of a rheostat which was connected in the field of one of the machines.

The voltage was read on a Kelvin electrostatic voltmeter which had been calibrated with an attracted-disc electrometer. The current was measured with a D'Arsonval galvanometer whose figure of merit was found to be 6.25×10^{-6} amperes.

[1] F. W. Peek, Jr., Dielectric Phenomena in High Voltage Engineering, p. 27.

[2] S. P. Farwell, "The Corona Produced by Continuous Potentials," A. I. E. E., November 13, 1914.

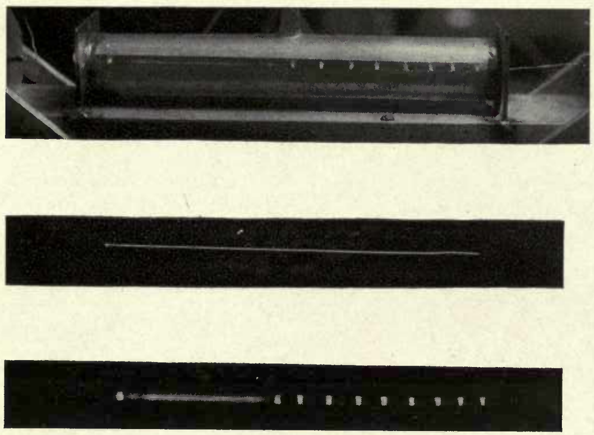

Fig. 1.

Steel Wire, 0.28 mm. Diameter.

	Corroded.	Polished.	Pressure.
No. 1.	Wire −,	2,000 volts,	30 mm.
No. 2.	Wire +,	2,300 volts,	40 mm.
No. 3.	Wire −,	2,300 volts,	40 mm.

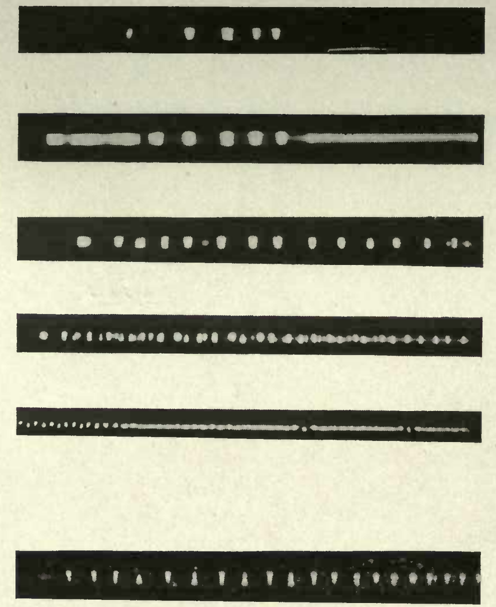

Fig. 2.

Steel Wire, 0.28 mm. Diameter.
All wires negative.

	Abrased.	Polished.	Corroded.	Pressure.
No. 4.		990 volts,		21 mm.
No. 5.		1,080 volts,		25 mm.
No. 6.		1,400 volts,		39 mm.
No. 7.		2,300 volts,		68 mm.
No. 8.		8,900 volts,		370 mm.

Steel Wire, 0.41 mm. Diameter.

	Abrased.	Polished.	Corroded.
No. 9.		1,650 volts,	30.2 mm.

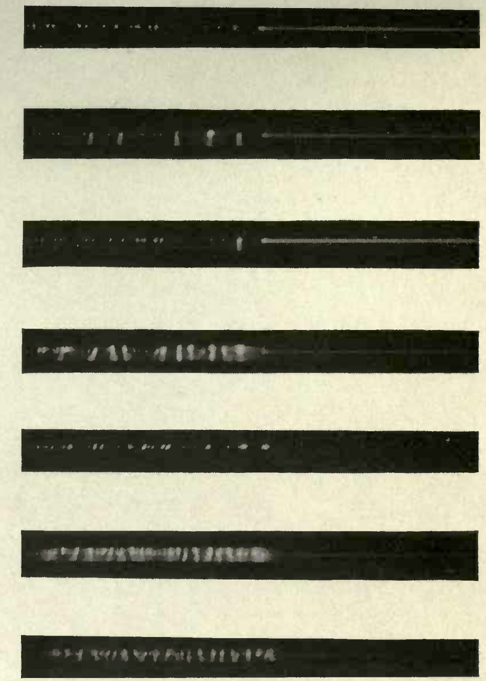

Fig. 3.

German Silver Wire, 0.65 mm. Diameter.

	Enameled.		Polished.
No. 10.	Wire +,	1,460 volts,	23.5 mm.
No. 11.	Wire +,	1,700 volts,	25.5 mm.
No. 12.	Wire +,	1,830 volts,	35.5 mm.
No. 13.	Wire +,	2,800 volts,	91.5 mm.
No. 14.	Wire −,	3,920 volts,	185.5 mm.
No. 15.	Wire +,	4,400 volts,	185.5 mm.
No. 16.	Wire +, arc in series,	4,450 volts,	185.5 mm.

SYLVAN J. CROOKER.

Preliminary Experiments.—Preliminary experiments were made using a steel wire the surface of which was polished over one half its length and corroded with nitric acid over the other half. When the wire was placed in the tube and corona made to form on it at low pressures, the effect of the surface condition was made evident at once. Nos. 1 and 3 in Fig. 1 show this experiment. The right half of the wire is polished and has the characteristic negative beads, while the left half is chemically corroded and only a soft glow appears there. This glow is different from the characteristic positive glow in that it is much greater in diameter and has a fuzzy appearance like eider down.

Of course it must be noted here that this condition is for a slightly higher potential than that at which the glow first appears and that the fuzzy glow eventually breaks into the beads upon raising the potential. However the beads on the corroded end do not have the sharp clear-cut appearance as those on the polished end, but are fuzzy and less well defined. The positive glow is also shown in Fig. 1, No. 2, under these same conditions, but it presents the same appearance for both parts of the wire.

The first experiment led to the trial of a wire whose surface was not only (1) polished and (2) corroded, but also (3) mechanically abrased. The differences existing here were also very striking and clearly shown at once. Fig. 2 contains photographs of the negative wire, the left end being abrased, the center polished, and the right end chemically corroded.

No. 4 shows the starting of the corona at low pressures and correspondingly low voltages. It will be seen that the beads start first on the polished surface (1), while the corroded surface (2) shows no glow and the abrased surface (3) has but a slight brush discharge on it. The beads on (1) are very large, clear, steady and quite evenly spaced.

No. 5 shows the effect of a slight increase in voltage where the glow now appears on surface (2) and the beads begin to form on surface (3). Gradually increasing the voltage and the pressure as well causes the glow to become brighter on (2), the beads to increase on (1) and (3). The beads on the abrased portion have a lateral movement, while those on the polished part are still very steady and clear.

With still greater increase in pressure and voltage it is possible to reach a condition where the whole length of the wire is covered with clear, steady and evenly spaced beads (see No. 6). Here it seems that the surfaces all act very nearly the same regarding the formation and building up of the corona discharge.

Now when the pressure is increased to 370 mm. and the voltage is increased to produce the discharge it is found that the corona starts first

on the abrased portion and that it is only on this part clear steady beads can be obtained (see No. 8). The beads on the corroded part are fairly well defined but they are in an agitated state, moving back and forth on the wire. Under these conditions it is found impossible to get steady beads on the polished part of the wire; instead of the clear beads there is a rather knobby glow on the wire, the condensations in which seem to be beads trying to form.

This reversal of the phenomena, as shown in Fig. 2, where the clear beads form on the polished surface at low pressures and on the abrased surface at high pressures, has been found to be a real one for steel wire. The corona starts first on the polished wire for low pressures and begins on the abrased or corroded wire at much lower potentials for high pressures.

An enameled german silver wire was fitted in the tube after one half of its length had been freed from the enamel and polished. At low pressures for the positive wire the characteristic glow would appear on the polished end. The enameled end would have several small star-like spots of light irregularly distributed over it appearing at points where the insulation had broken down. Keeping the wire positive and increasing the voltage caused very bright "streamers" of purple light to shoot out from a few of these small stars. At higher pressure and higher

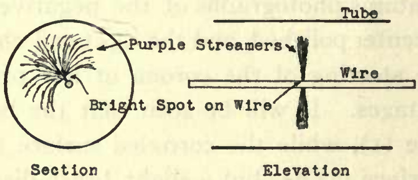

Fig. 4.

voltage these streamers increased greatly in number, the glow spreading out into a thin fan-shape. This fan would slowly oscillate or rock back and forth about the bright spot on the wire as a center. Between these fans a hazy fog-like glow was everywhere present. Upon placing an arc in series with the wire and the tube this fog would disappear and the fans would become more sharply defined and more steady.

For the wire negative (see Fig. 3, No. 14) it was impossible under any conditions to get the characteristic negative beads. Neither could a glow be produced on the polished end, the only discharge present was on the enameled end similar in appearance to the small stars for the positive wire. However for the negative wire the stars were intensely bright and in slight movement. Fig. 3 shows the appearance of the discharge from the enameled wire when both positive and negative. Fig. 4 gives

details of the structure of the positive purple fans. For the enameled wire negative the starting potential was much lower than for the opposite polarity.

Figs. 1 and 3 suggest that the starting point of the corona and the characteristic curves depend on the surface conditions. In order to test this suggestion the following experiments have been performed.

II. Visual Corona and Starting Potentials.

Many characteristic curves were obtained for different sizes of copper wire where the surfaces were polished and abrased and many more characteristic curves were taken, using wires of copper, steel, aluminum, and silver where the surfaces were polished, abrased or roughened, and chemically corroded or oxidized. The more striking results will be given in the following paragraphs.

Preparation of Surfaces.—For the polished surfaces care was taken in choosing wires without kinks or surface scratches. These wires were polished with fine emery cloth and finished with chamois and jeweler's rouge just before placing in the tube.

The abrased surfaces were prepared by rolling the wire in emery powder between two hard plane surfaces. Care was taken to have the surface abrased uniformly over the whole length.

The corroded surfaces were prepared by different methods. The surface of the steel wire was corroded by dipping in a solution of nitric acid, a black surface resulting. The aluminum wire was corroded by allowing it to remain in a solution of sulphuric acid for a few days. The result was a thin white coating. For copper it was necessary to oxidize the surface by passing a heating current through the wire in the presence of oxygen. Since large quantities of ozone are produced by the corona discharge the silver wire was coated with a layer of silver peroxide by allowing the corona to play on the wire for some time.

The phenomena are very complicated. Their description will be carried out according to the surface condition of the wire, and for each individual condition three pressures will be considered.

Wires Polished.—The general appearance of the corona is the same for all polished positive wires, and differs but slightly for negative wires at the different pressures. At pressures of about 50 mm. when the potential is brought up to the glow potential, wire positive, a very faint flashing glow is seen over the whole length of the wire, which becomes uniform and steady as the potential is raised slightly. The potential may be carried up to the arcing point without changing the general appearance of the uniform glow. The only noticeable change is an increase in the brightness of the bluish glow.

For pressures of 50 mm. and negative wire, the first appearance of the corona is a flashing glow, similar to that for positive wire, but of much greater diameter and brighter. Increasing the potential causes this glow to remain steady on the wire, becoming uniform and very bright. Very little current flows until a stage is reached not far above the starting point, where the bright uniform glow breaks into large clear characteristic negative beads. From this point on the current increases rapidly with the potential. As the potential is increased the beads increase in number but remain large and well defined, this will be discussed more fully later on.

For the polished surfaces and pressure of 50 mm. the negative corona on copper begins at a lower potential than the positive. Corona appears at the same potential for both polarities in the case of steel, but for aluminum and silver the positive glow begins at the lower potential. This

TABLE I.

COMPARISON OF STARTING VOLTAGES FOR DIFFERENT SURFACES AND WIRES.

All wires about 0.41 mm. diameter.

Polished			Abrased			Corroded		
Press. mm.	Wire – Volts.	Wire + Volts.	Press. mm.	Wire – Volts.	Wire + Volts.	Press. mm.	Wire – Volts.	Wire + Volts.
Copper.								
50	1,700	1,780	53.2	1,680	1,820	50.3	1,650	1,660
252	2,650	2,600	253	2,550	2,800	250	2,010	2,500
731	6,010	5,760	743	5,600	6,200			
Steel.								
51.6	1,710	1,710	52.2	1,690	1,740	52.3	1,750	1,700
252.4	2,600	2,600	253.2	2,770	2,770	252	2,550	2,710
727.6	5,660	5,960	736	4,560	5,830	739.4	4,810	5,760
Aluminum.								
50	1,760	1,720	52	1,660	1,800	51.9	1,240	1,690
251	2,820	2,900	251.5	2,490	2,900	252	2,370	2,660
741.1	5,880	6,180	741	5,010	5,800	745.3	4,680	5,880
Silver.								
53.2	1,850	1,820	52.3	1,730	1,740	52.5	1,850	1,780
252.1	3,150	3,050	252.2	2,600	2,900	252.2	3,150	3,000
744.8	4,210	6,130	743.2	5,060	5,850	746	5,760	6,320

is shown by Table I., which contains the starting potentials for the different metals and different surface conditions. Table I. shows no general law. With the exception of the silver wire at a pressure of 746

mm. the starting potential for the corroded wire is smaller for both polarities than for the polished wire. For the negative abrased wire the starting point is in general lower than for the polished wire with only two small exceptions. With the exception of silver the starting point of the abrased positive wire is higher than that of the polished wire. With increasing pressure the differences involved by abrasion and corrosion diminish. The largest influence is found for aluminum wire, negative corroded at 51 mm.

For pressures of about 250 mm. the glow for wires positive is the same as before, being uniform and increasing in brightness as the potential increases. For wires negative and polished it was almost impossible to break the glow up into clear-cut beads at this pressure. With increasing potential the glow would become brighter and would condense at certain ill-defined points apparently attempting to form beads, but these condensed regions would be in rapid motion back and forth along the wire.

For atmospheric pressure, wires polished and positive, the glow would appear faint but uniform and would increase in brightness as the potential was increased. For negative wires a faint flashing glow would appear at break-down potentials increasing in brightness with the potential increase. A very few scattered beads would at times be formed, but they would be small and unstable having very rapid lateral motion. This motion would increase in amplitude and speed with increasing voltage. Clear cut beads over the whole wire was impossible here as in the last case.

Wires Mechanically Abrased.—With wire surfaces mechanically abrased or roughened and pressure of 50 mm. the positive glow begins with faint flashes as in the case of the polished surfaces, the glow becoming steady, uniform and increasing in brightness as the potential is increased. The starting glow voltage is in general higher than for the positive polished wires, and is also higher than for the abrased negative wires. For wires abrased and negative the corona begins with bright flashes of a fuzzy glow, part of which might have one or two large flashing beads. This flashing glow seemed to pulsate in synchronism with the impulses of the driving machinery. A slight potential increase above the first noticeable glow would cause the glow to break into well-defined beads which would soon become steady and clear, increasing in number with a potential increase. The negative starting voltage for abrased wires is lower than for the polished surfaces.

For wires abrased and pressures of 250 mm. the positive visual glow is the same as before. The positive starting potential is in general higher than for the negative abrased and also positive polished surfaces. The

negative glow voltage causes very faint "spears" or small brushes of light to flash out from sharp points here and there on the rough surface. These spears increase in size and number with increased potential, some being much brighter than others. As the potential is increased these spears unite into definite, clear beads which at times may be very steady and at other times may have more or less violent lateral movements. The negative starting voltage for abraised surfaces is much smaller than for the polished surfaces.

At atmospheric pressures the positive glow on the abrased wire surfaces usually begins with a few small flashing purple streamers or brushes extending from the wire almost to the tube. These streamers are similar in appearance to the positive fans and streamers emitted from the surface of the enamel covered wire, see Figs. 3 and 4. These streamers increase in brightness and are accompanied by soft glow as the potential is increased. After a certain increase has taken place in the voltage these streamers disappear only the uniform glow remaining and increasing in brightness.

For the abrased negative wire at atmospheric pressure the corona starts

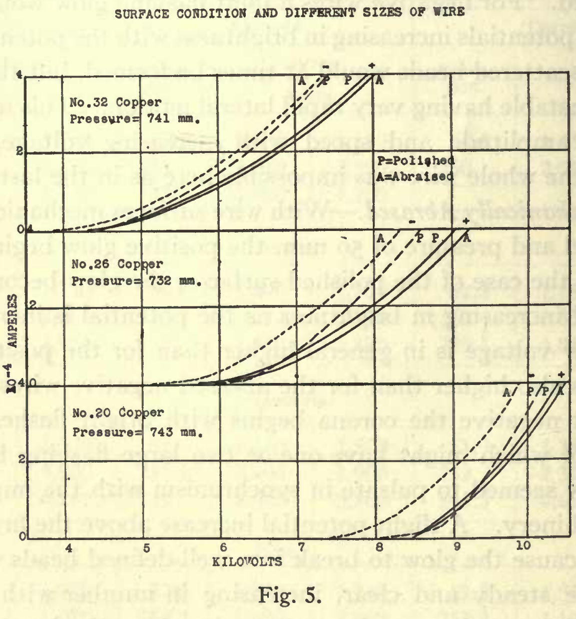

Fig. 5.

with small flashing spears the same as for the abrased wire at 250 mm. These spears increase in number very rapidly with an increase in voltage, some of them collecting, so to speak, into small bright beads and then breaking up again. As the potential is still more increased the beads

become more steady and definite, so that at times the abrased wire may be covered with many small, bright, steady and evenly spaced beads.

Chemically Corroded Surfaces.—The positive visual corona for corroded surfaces is essentially the same for all pressures as has been described for the abrased surfaces. At low pressures it begins with a faint flashing glow which becomes steady and uniform, increasing in brightness. At pressures of 250 mm. the appearance is the same as above, and for atmospheric pressure the corona may start with the small purple brushes or fans and an accompanying glow, the fans soon disappearing and the glow becoming uniform and increasing in brightness. The positive glow generally begins at lower voltages for the corroded surfaces than for the polished.

The negative visual corona for the corroded surfaces is likewise similar to that for abrased surfaces at the different pressures. Clear cut and steady beads are obtained at the lower pressures but are not as stable for the higher pressures. In general the negative starting voltage is lower than for polished surfaces.

III. Characteristic Curves for Different Wires and Surfaces.

Varying the Radius of the Wire.—The curves in Fig. 5 are taken for different sizes of copper wire. They show that the effect of abrasion in general lowers the starting point for copper wires at atmospheric pressure. The negative abrased curves are widely displaced from the polished ones, showing that more current flows in the corona discharge for the same voltage for wire abrased than for the smooth wire. The positive abrased curves quickly cross the polished ones and then continue to rise slightly displaced, less current flowing for the same potential abrased than for polished. Thus the abrased surface has the effect of restraining the flow of the positive current.

The effect of abrasion is much greater in the case of the negative current. The curves also show that this effect is greater for the larger sizes of wire, which might be expected. The higher starting potentials for the larger-sized wires is also evident.

The negative current builds up very slowly at first on the polished surface but finally reaches a point where it builds up much faster than the positive; at this point the beads are formed. The starting voltage for the abrased surface negative is much lower than for polished negative. The characteristic curve of the abrased wire is a smooth rising one eventually crossing the polished negative curve for large current values. This same phenomenon has been observed for different metals.

Different Surface Conditions for the Same Metal.—Fig. 6 gives the char-

acteristic positive and negative curves for aluminum wires at about 50 mm., showing the effect of the three surface conditions; namely, polished, abrased and corroded. The starting positive wire voltage for the smooth surface is slightly lower than that of the negative, but the curves cross low, the positive current building up quite slowly with increased potential, while the negative curve is almost a straight line rising

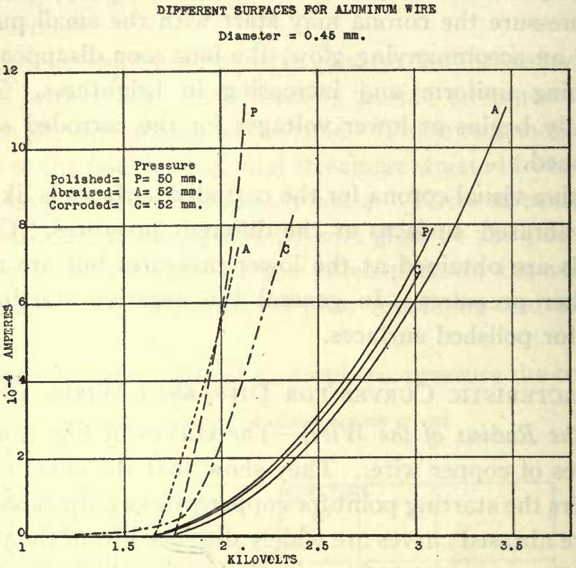

Fig. 6.

very rapidly. The positive starting potential is higher for the abrased surface than for the polished, while that for the negative abrased surface is lower. The negative polished and abrased surface curves cross but the positive do not. For the corroded surface the positive glow voltage is about the same as for the polished surface, the curve for the former condition becoming displaced shortly, less current flowing for the same voltage. The negative starting potential is very much lower in the latter case than that for the polished surface, but crosses at a low current value and rises to the right, less current flowing for the same potential.

Thus it is seen that the surface condition has a very marked effect on the starting point of the corona as well as on the characteristic curves. All the wires were about 0.41 mm. in diameter. In general the abrased surface has the effect of lowering the starting potential for negative wire and raising it for positive wire. The starting point for both positive and negative in the case of corroded wires is in general lower than for the polished surfaces, but the corroded surface characteristics behave in

rather an erratic manner, sometimes being displaced in one way and sometimes in the opposite.

Table II. gives a comparison between the corroded and polished wire characteristics for both positive and negative at different pressures.

Table II.

Comparing Corroded with Polished Wire Characteristics.

Wire.	Press.	Starting Pot.	Corroded Surface Characteristic.
		Copper.	
–	50.2	Lower	Raised.
+	50.4	"	"
–	250.0	"	Crosses high.
+	250.8	"	Raised.
		Steel.	
–	53.2	Higher (press. diff.)	Corsses high.
+	52.4	Lower	" "
–	252.0	"	" low.
+	252.4	"	Lowered.
–	739.4	"	Crosses high.
+	739.4	"	" low.
		Aluminum.	
–	51.9	Lower	Crosses low. (For instance see Fig. 4.)
+	51.9	"	" "
–	252.0	"	" midway.
+	252.0	"	Raised.
–	745.3	"	Crosses midway.
+	745.4	"	Raised.
		Silver.	
–	52.5	Same	Lowered.
+	52.5	Lower	"
–	252.2	Same	Crosses low.
+	252.2	Lower	" midway.
–	745.8	Higher	Lowered.
+	746.1	"	"

Different Metals of the Same Radius and Surface Condition.—Farwell[1] by electrolytic processes covered the surface of a wire with different metals to determine their effect. He observed slight discrepancies but attributed them to experimental errors and concluded with Whitehead[2] that the formation of the corona is independent of the material of the wire.

Table I. compares the starting voltages for wires of the same size but

[1] S. P. Farwell, "The Corona Produced by Continuous Potentials," A. I. E. E., November 13, 1914.

[2] Whitehead, "The Electric Strength of Air, I.," A. I. E. E., July, 1910.

of different kinds of metal for different surfaces and pressures. Curves in Fig. 7 show a comparison between the characteristics of different metals. Very marked differences are evident in the characteristic curves, showing directly that the metal itself has a part to play in the corona formation. The positive and negative characteristics, especially

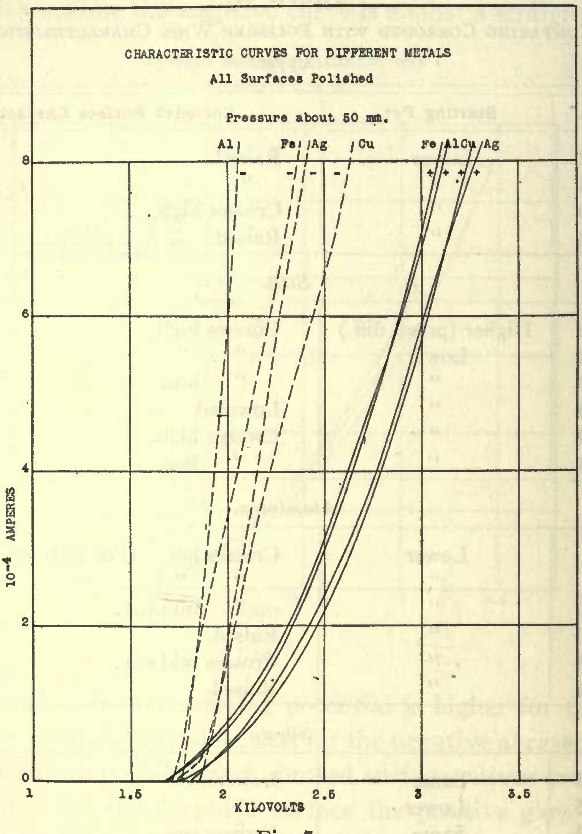

Fig. 7.

for the case of aluminium, become widely separated for large currents, the curves for the other metals separate at different rates for increasing current values, but in such a manner that each metal behaves in its own characteristic way.

Slight differences in the starting points for the different metals were noticed; these differences however are of such a nature that they cannot be explained as being experimental errors. Steel and copper seem to have about the same starting point, while that for aluminum is a little higher and silver has a value still greater. The different metals not only affect the behavior of the characteristic curves but also the starting points of the corona glow.

The Effect of Ozone on the Corona.—The presence of ozone has a definite effect on the appearance of the corona as well as on the characteristic curves. If ozone is present in the corona tube in any quantity the negative beads do not form quite as distinctly as they do when a stream of air is passing through the tube, carrying the ozone away. The effect on the negative characteristic curve is very slight, displacing it a little to the right, such that less current can flow for the same potential (see Fig. 8). For the positive characteristic the effect is somewhat larger

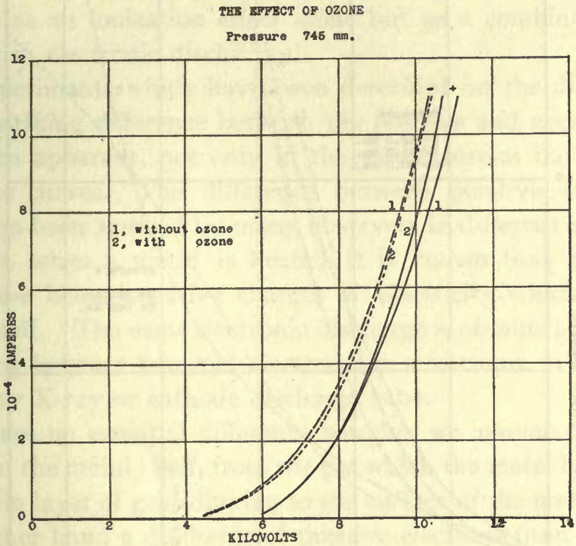

Fig. 8.

and in the opposite direction, the curve being displaced to the left, showing that more current for the same potential flows through the tube with ozone present than does in its absence. At atmospheric pressure ozone is formed quite rapidly but its effect is not large. At lower pressures with less gas present the formation of ozone is very much less and its effect on the corona is proportionately smaller.

The presence of ozone does not explain the differences in the characteristic curves for the different metals, unless it is a secondary effect between the wire and the ozone.

Formation of the Negative Beads.—The formation and number of the negative beads depends not only on the pressure and potential, but also on the surface condition and the material of the wire. (See Farwell on Material of Wire.) Fig. 9 shows the relations between the number of beads and the current for different surfaces of copper wire. The current per bead is larger for the abrased and corroded surfaces than for the

polished surface, assuming the whole current to be carried by the beads. For an increase in pressure it is also seen that the current per bead is much less, but the beads are smaller in size. However, for the higher pressures it takes a larger voltage to produce the same number of beads. For the lower pressures the beads have about the same degree of stability

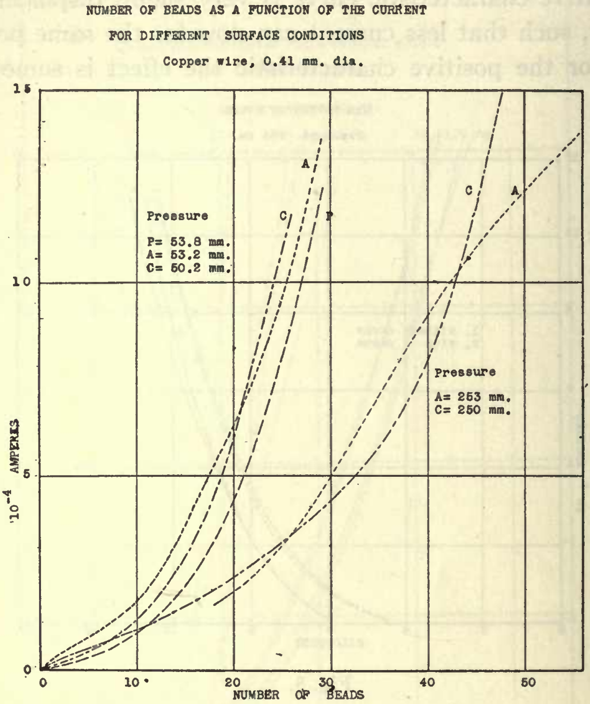

Fig. 9.

for all the different surfaces, while for higher pressures the beads are more stable on the abrased or corroded surfaces than on the polished, it being almost impossible to get definite beads on the polished wire for atmospheric pressures.

The number of beads increases rapidly with increasing voltage. Here again the effect of the materials is compared. For the production of the same number of beads it takes in general a greater voltage on the steel than on the copper and aluminum wires.

IV. Theoretical.

Electronic and Ionization Effects Combined.—The theories by Townsend[1] and Bergen Davis,[2] which have been proposed to explain the corona phe-

[1] Townsend, "A Theory of Glow Discharges from Wires," Electrician, June 6, 1913.

[2] Bergen Davis, A. I. E. E., April, 1914.

nomena, have been based upon the assumption that only an ionization of the gas takes place. These theories have explained some parts of the observed phenomena very well, but as for other parts it is impossible to produce a complete explanation by this one assumption. It is conceded that the ionization effect has a great deal to do with the corona action but it is not possible that there are other effects which are working in conjunction with this one.

In the following an attempt will be made to explain the corona phenomena not as an ionization effect alone but as a combined action of ionization with electronic discharges.

In the experiments which have been described on the direct current corona the striking difference between the positive and negative corona is everywhere apparent, not only in the visual corona but also in the characteristic curves. The difference between positive and negative electricity has been noticed by many observers in different experiments. For example, when a metal is heated it is known that electrons are shot off, these being negative charges of electricity which come from the metal itself. The same electronic discharge is obtained when a large force is acting between two cold electrodes in a vacuum. The example is seen in any X-ray or cathode discharge tube.

It will make no essential difference whether we assume the electrons to come from the metal itself, from the gas which the metal has absorbed, or from a thin layer of gas adhering to the surface of the metal.

On the other hand a discharge of positive electrons from a metal has never been observed. It always requires the presence of a gas to produce the positive charges of electricity. Experiments have also shown that these positive charges are atomic in size and hence are to be considered as positive ions.

The assumptions which are made in this theory then, are that there is a combined action of electronic discharge from the metallic surface along with ionization in the gas. In some cases the electrons will predominate in determining the character of the phenomena, while in others the ionization may be the determining factor.

Wire Surfaces Bare and Insulated.—It might ordinarily be supposed that to insulate the wire would increase the starting potential for the corona and cut down the loss. However, just the reverse of this was observed in Fig. 3 where part of the wire is covered with insulation and part bare and polished. It would seem almost like a paradox to say that insulation increases the corona loss, but it is found possible to explain this phenomenon by assuming electronic discharge from the metal and ionization in the gas.

Fig. 10 represents the conditions in this experiment. A potential V is impressed between the wire and the tube.

R_1 = the radius of the wire,

R_2 = the radius of the insulation, and

R_3 = the inside radius of the cylindrical tube.

The electric force E at the surface of the wire which is covered with insulation is given by the equation,

$$Ek2\pi R = 4\pi e,$$

where k = dielectric constant, R = point in which the force is being measured, and e = charge on the surface of the wire per unit length.

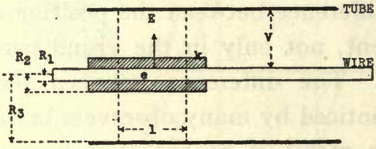

Fig. 10.

For a unit length of the surface,

$$E = \frac{4\pi e}{k2\pi R} = \frac{2e}{kR}. \tag{1}$$

To calculate the potential or the work done in carrying unit charge through the distance dR, multiply by dR,

$$EdR = \frac{2e}{k}\frac{dR}{R}.$$

The work done in transporting the charge from R_1 to R_2 is

$$\int_{R_1}^{R_2} EdR = \frac{2e}{k}\int_{R_1}^{R_2}\frac{dR}{R} = \frac{2e}{k}\log\frac{R_2}{R_1}. \tag{2}$$

Similarly, the work done through the distance R_2 to R_3, where $k = 1$, is

$$\int_{R_2}^{R_3} EdR = 2e\int_{R_2}^{R_3}\frac{dR}{R} = 2e\log\frac{R_3}{R_2}. \tag{3}$$

The potential

$$V = \int_{R_1}^{R_2} EdR + \int_{R_2}^{R_3} EdR = 2e\left(\frac{1}{k}\log\frac{R_2}{R_1} + \log\frac{R_3}{R_2}\right), \tag{4}$$

$$e = \frac{V}{2\left(\frac{1}{k}\log\frac{R_2}{R_1} + \log\frac{R_3}{R_2}\right)} \tag{5}$$

$$E = \frac{2e}{kR} = \frac{V}{kR\left(\frac{1}{k}\log\frac{R_2}{R_1} + \log\frac{R_3}{R_2}\right)} = \frac{V}{kR\log\left(\frac{R_2}{R_1}\right)^{\frac{1}{k}}\frac{R_3}{R_2}}. \tag{6}$$

The capacity when the wire is insulated is readily calculated since,

$$C = \frac{e}{V} = \frac{1}{2\left(\frac{1}{k}\log\frac{R_2}{R_1} + \log\frac{R_3}{R_2}\right)} \tag{7}$$

Then when air only is the intervening medium, $k = 1$ and $R_2 = R_1$, therefore,

$$C_1 = \frac{1}{2\log\frac{R_3}{R_1}} \tag{8}$$

and from (5),

$$e_1 = \frac{V}{2\log\frac{R_3}{R_1}} \tag{9}$$

By comparing (7) and (8) it is seen that the capacity is increased by placing insulation on the wire and we can therefore conclude that with the same potential difference the charge e on the surface will also be increased.

The force necessary to draw unit electric charge out from the metal when it is insulated is expressed by the relation

$$F = \frac{2\pi\delta^2}{k}, \tag{10}$$

but

$$\delta = \frac{e}{2\pi R_1},$$

then

$$F = \frac{e^2}{k2\pi R_1^2}$$

and from (5)

$$e^2 = \frac{V^2}{4\left(\frac{1}{k}\log\frac{R_2}{R_1} + \log\frac{R_3}{R_2}\right)^2};$$

therefore

$$F = \frac{V^2}{k8\pi R_1^2\left(\frac{1}{k}\log\frac{R_2}{R_1} + \log\frac{R_3}{R_2}\right)^2}. \tag{11}$$

Since the insulation is very thin R_2 is very nearly equal to R_1, and

$$\frac{1}{k}\log\frac{R_2}{R_1} = 0,$$

approximately and (11) becomes,

$$F = \frac{V^2}{k8\pi R_1^2\left(\log\frac{R_3}{R_1}\right)^2}. \tag{12}$$

When there is no insulation on the wire, $R_2 = R_1$ and only air remains between the electrodes, $k = 1$, and (11) reduces to,

$$F_1 = \frac{V^2}{8\pi R_1^2 \left(\log \dfrac{R_3}{R_1}\right)^2} \tag{13}$$

F and F_1 differ only by the constant $1/k$ so it is easily seen that the force F necessary to pull the electrons from the wire surface which is covered with insulation is much smaller than F_1, the force necessary to draw the electrons from the free surface after the insulation has become punctured. Therefore when the wire is negative, as is the case in No. 14, Fig. 3, there will be glow on the insulated side appearing at points where the insulation has broken down and there will be no glow on the polished surface. The discharge in this case is essentially electronic, the spots of light which appear are intensely bright and the potential at which glow begins is very much lower than when the wire is positive.

When the wire is charged positively as in Fig. 3, No. 15, there appears a faint uniform glow on the polished surface. This is the characteristic positive glow which has a pale blue color. This may be explained as being essentially an ionization effect. That is, the wire being charged to a certain positive value has force enough to split up the gas molecules by collisions in its immediate neighborhood and when the energy is large enough light is emitted. The negative particles are attracted to the wire while a layer of positive ions collect at the wire surface.

On the enameled end of the wire the density of electricity is larger per unit length. The streamers or fans of purple light approach the appearance of the direct current arc both in form and color, so we might say that these streamers are negative ions moving toward the positive wire with a great velocity. An analogy to this brush discharge phenomenon would be the stream lines of air entering small holes in a pipe carrying vacuum. The particles of air which are drawn to the pipe with increasing velocity are analogous to the negative ions which are drawn to the wire with velocity increasing as the wire is approached. These purple brushes have been noticed at different times for bare wires when positively charged. They seem to come especially from irregularities or points on the wire where there would be a great surface density.

The starting potential for corroded surfaces has in general been found to be much lower than for polished surfaces. An explanation is easily found by considering either one of two effects. Either the size of the wire is slightly reduced by corrosion, enabling the corona to start at lower voltages or the corroded surface acts as an insulator giving the same

condition as has been explained for the phenomena in Fig. 3. The difference in starting potentials for corroded and polished surfaces is in general so large that the former explanation is hardly feasible, since the size of the wire could not have been greatly reduced. The latter seems to give the best explanation, since it is known that most of the oxides when dry are good insulators, and it would be possible with such an insulating layer to get a large difference in the starting potentials.

The Negative Beads.—The negative beads may be considered as being unstable in two senses. First, they move back and forth along the length of the wire, and, second, they give rise to oscillations. For instance it is known that a gas column or stream of electrons as in the Poulson arc is very unstable and gives rise to high frequency oscillations.

At the very beginning of the negative corona the glow covers the whole wire. A certain amount of energy is stored up in this layer which is in an unstable condition and easily breaks up into the characteristic beads. This is similar to a film of water covering a wire or string. The film will be uniform until a certain point of instability is reached when it will break up into drops or beads. This breaking up of an original uniform layer into beads has been observed over and over again. As soon as a bead is formed a large current starts in that region which heats the gas and the metal at that point. A thin metal wire may easily melt. If the temperature rises and the potential difference decreases at these points, the metal will be oxidized and the discharge will take place at a different point, causing the beads to move backward and forward along the wire.

Moreover the beads assume the shape of a fan whose plane is perpendicular to the wire, so that in these planes the temperature will be higher than in the neighboring regions. This will give rise to an unstable temperature distribution which will cause the beads to move along the wire. This effect is more pronounced when the wire is in a vertical position.

This instability is also shown by the fact that the pressure increase due to ionization when the wire is negative is very erratic and cannot be measured accurately, and by the fact that the field in the tube cannot be investigated by a third sounding-electrode, since very irregular results are obtained due to the presence of the beads, while the positive wire gives very regular results which can easily be repeated and show a marked distortion of the field between the wire and the cylinder.

There are several other cases in which the negative elctricity escapes from surfaces; for instance, in the mercury arc the glow from the negative terminal does not come from the whole surface of the electrode but from a bright spot on the surface of the mercury which moves about irregularly. Another case would be that of the ordinary carbon arc under certain

conditions when the negative end of the flame moves about, and still another, Dr. Knipp's cylindrical cathode.[1] Indeed the negative bead resembles the arc in several respects; it may be called a small arc which by increasing the voltage gradually goes over into the more definite arc.

The beads represent in the second place a more or less unstable discharge in so far as oscillations are very easily set up. S. P. Farwell has shown that a small spark gap in series with the corona tube gives rise to oscillations in the electric circuit. Bennett[2] has shown that oscillations arise readily in the negative part of the corona for alternating currents. The arc, for instance the Poulson arc, is a transformer of direct current into alternating current of a very high frequency.

The beads are always brighter and steadier at low pressures that at high pressures. At low pressures the electronic discharge from the metal predominates over the ionization by collision in the gas. With increasing pressure the ionization by collision becomes more and more important, the beads become smaller and more numerous.

For a certain pressure and potential difference beads will appear on the abrased, polished and corroded surfaces of a steel wire in exactly the same way (see fig. 2, Nos. 6 and 9). This happens between the pressures of 30 and 40 mm. At a lower pressure (25 mm.) and a smaller potential difference beads appear only on the polished part, while a more or less uniform glow covers the corroded and the abrased portions; the surface irregularities on the corroded and abrased parts giving rise to very many overlapping beads, forming a soft glow. There are, as it were, too many but too weak opportunities for the formation of well-defined beads. For still lower pressures and potential differences the original glow covers only the polished portion first and it is only on that part that the clear beads will form.

Returning to the pressure, 30 to 40 mm., where the beads are evenly distributed over the whole wire, and increasing the pressure and the potential difference, then the number of beads increases. They become unsteady and fuzzy especially along the corroded and polished parts and finally with still higher pressures the beads are only well defined on the abrased part, where they probably are fixed by rough surface irregularities which act like small lightning rods. During all of these changes of the negative corona the positive glow remains perfectly constant, forming a well-defined uniform bluish glow along the wire.

It has already been shown that one should expect for corroded surfaces a smaller starting voltage than for the polished wire for both polarities.

[1] Dr. C. T. Knipp, Science, May, 1916.

[2] Bennett, Trans. A. I. E. E., Vol. 32 (1913).

For the negative abrased wire the starting voltage also is smaller than for the negative polished wire, a result which is evident. As clear beads are formed for the abrased wire at higher pressures one should expect that the negative characteristic curve is higher than that for the polished wire and that is actually the case (see Fig. 2, No. 8, and Fig. 5). On the other hand if at low pressures bright beads are formed on the polished wire one should expect the current to be larger than for the abrased and corroded wire and this also is the case. (Compare Fig. 2, No. 4, and Fig. 6.) Bright beads are always accompanied by a large current. Corrosion and abrasion have little influence on the positive characteristics.

Conclusions.

1. The surface conditions as well as the metal itself has an effect on the starting voltage and on the characteristic curves.

2. The number and brightness of the negative beads depend in a very complicated way on the surface conditions.

3. A thin layer of insulation on the wire renders the escape of negative electricity easier. This paradox has been explained.

4. The formation of beads and their instability has been explained on the assumption that the current is due to an emission of electrons from the surface of the metals and due to ionization by collision. Most of the complicated phenomena have been explained by this assumption.

An expression of thanks must be given to Prof. A. P. Carman through whose kindness facilities for these experiments were provided, and to Dr. Jakob Kunz, whose suggestions and assistance have proven a constant source of help throughout the work.

Laboratory of Physics,
University of Illinois,
Urbana, Illinois,
May 15, 1916.

[*Reprinted from* SCIENCE, *N. S., Vol. XLIX., No. 1271, Pages 450–451, May 9, 1919*]

TO CUT OFF LARGE TUBES OF PYREX GLASS

ON a number of occasions I have heard the remark from instructors in physics and chemistry, who do most of their own glass blowing, that they are unable to "cut" off squarely large tubes of pyrex glass. Small tubes, up to about 20 mm. in diameter, yield readily to the usual file mark.

A well-known method for cutting large tubes of common glass is to make a file scratch round the tube, apply one turn of an iron wire held taut, and then heat the same to redness by an electric current.

This method, however, without modification, fails when attempting to cut pyrex tubes. The glass will simply not crack, and if the heating is pushed the hot wire usually sinks into the glass and finally fuses under the intense heat.

I was surprised recently to find that if the iron wire is replaced by a nichrome wire, say, of no. 14 or 16 gauge, the tube may be *cut* off by the incandescent wire in the same manner that a cake of soap is cut in two parts by means of a string.

To insure success proceed as follows: Take a length of about one foot of nichrome wire, connect it up to a D. C. (or A. C.) dynamo current and include an adjustable tin resistance (for the current required must necessarily be large). The wire is held under tension by pulling on it with a pair of pincers, as shown in Fig. 1. Care must be taken not to let the two parts of the wire touch at *A*. When all is in readiness, turn on the heating current and adjust same by means of the tin

resistance until the wire glows a white heat. If now a blast from a hand torch be allowed to play on the wire and glass the tube may be cut as shown in Fig. 2. Be careful not to let the flame strike the glowing wire where it is not in contact with the glass for the extra heat will burn it. The object of the blast is

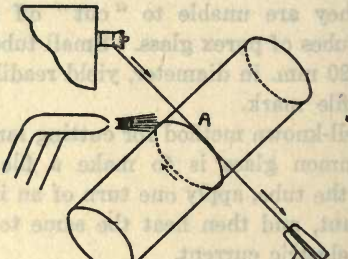

Fig. 1.

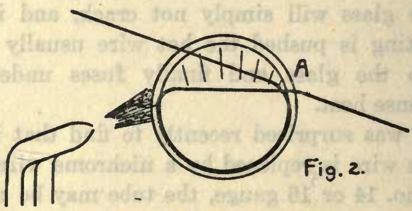

Fig. 2.

to aid in softening the glass, and also to distribute the heat along the tube and thus prevent the freshly cut edges from checking due to the otherwise intense local heating. The burr of glass that results from the cutting may be removed by a file or on the grindstone.

Recently the neck of a twelve-liter pyrex Florence flask was cut off with the greatest ease. The diameter was about 60 mm., and the wall thickness about 2.5 mm.

CHAS. T. KNIPP

UNIVERSITY OF ILLINOIS

[Reprinted from the PHYSICAL REVIEW, N. S., Vol. IX, No. 3, March 1917.]

TOLMAN'S TRANSFORMATION EQUATIONS, THE PHOTO-ELECTRIC EFFECT AND RADIATION PRESSURE.

BY S. KARRER.

R. C. TOLMAN uses the following transformation equations when applying his principle of similitude,

$$l' = xl; \quad t' = xt; \quad e' = e; \quad m' = m/x; \quad S' = S.$$

P. W. Bridgman has pointed out that these equations are a particular case out of a large number of possible transformations which follow from the principle of dimensional homogeneity. Tolman has shown that, in many instances, the application of the above equations leads to results in accord with experiment. We wish to point out two important cases where the transformation equations give results which are in accord with present knowledge.

In the first place, let us assume we know from experiment that the kinetic energy E of the electrons emitted from a metal under the influence of light, minus a constant E_0 is only a function $f(n)$ of the frequency n of the incident light, and is independent of the intensity of the light and of the temperature and special properties of the metal. That is, assume

$$E - E_0 = f(n).$$

Then from the equations above it follows that

$$E' - E_0' = \frac{E - E_0}{x} \quad \text{and} \quad n' = \frac{n}{x},$$

$$E' - E_0' = f(n'),$$

$$\frac{E - E_0}{x} = f\left(\frac{n}{x}\right),$$

$$E - E_0 = xf\left(\frac{n}{x}\right) = f(n),$$

x being arbitrary, this functional equation must hold for every value of x. Its solution will be of the form,

$$f(n) = hn,$$

where h is a constant, hence

$$E - E_0 = hn,$$

which is Einstein's photoelectric equation. h must be determined by experiment; the measurements of Millikan give strong support to the correctness of this equation. And further, his results show that h is identical with Planck's radiation constant. From Tolman's point of view this is a very interesting result, but from the standpoint of Bridgman it is a necessary condition that the application of Tolman's equations give a correct result.

It is important to notice that Tolman's equations do not give any definite result in the case of the relation between the photoelectric current and the intensity of the light incident on the metal.

In the second place, we will apply the transformation equations to get the relation between the pressure exerted upon a body by the radiation incident upon it and the density of the radiation. Assume, experiment shows that the pressure p depends only on the energy density E of the radiation, then $p = f(E)$, and since

$$p' = \frac{p}{x^4} \quad \text{and} \quad E' = \frac{E}{x^4},$$

$f(E)$ must satisfy the following functional equation,

$$p = x^4 f\left(\frac{E}{x^4}\right) = f(E).$$

That is

$$p = f(E) = kE,$$

where k is a constant. As is well known, experiment shows that the radiation pressure is proportional to the energy density of the radiation.

Summary.

Tolman's transformation equations lead to Einstein's law of the photoelectric effect if it is assumed that the kinetic energy of the electrons emitted from a metal, minus a certain constant, is only dependent upon the frequency of the incident light.

The equations also lead to the correct relation between the pressure of radiation and the density of the radiation.

Laboratory of Physics,
University of Illinois,
January, 1917.

[Reprinted from the Physical Review, N.S., Vol. IX., No. 4, April, 1917.]

AN IMPROVED HIGH VACUUM MERCURY VAPOR PUMP.

By Chas. T. Knipp.

THE diffusion pump of Gaede[1] has stimulated a number of investigators in this country to enter the field of pump design with the result that several improvements involving new principles have been published. In a recent number of the PHYSICAL REVIEW Langmuir[2] describes an improved mercury vapor pump "characterized by its extreme speed and the high degree of vacuum attainable." The writer of this note being interested for a number of years in the production of high vacua also seized upon this opportunity to aid, if possible, in simplifying the means by which vacua are produced in the research laboratory and submits the following design made wholly of glass as an improved high-vacuum high-speed mercury vapor pump.

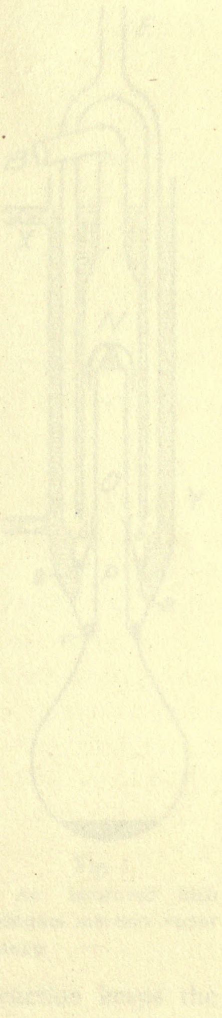

Fig. 1. All glass improved high-speed mercury vapor pump.

The pump complete, except the usual mercury vapor trap, is shown in the figure, which is one third full size. The bulb to be exhausted and trap are fused to B while the tube E is attached to the supporting or rough pump. The mercury vapor rising from the lower bulb, which is heated in a sand or heavy oil bath, streams up through the short tubes P and O and is deflected downward through an annular throat by the umbrella N. The issuing mercury vapor at once condenses on the water-cooled surface of the enveloping tube and, as Langmuir[3] pointed out, the gas that comes from B is forced mechanically downward from the lower edge of N along the cooled surface of the condensing chamber. This accumulated gas is removed through the lateral tubes DA which unite at the top and form the exhaust tube E, all being enveloped by the water jacket XY, as shown in the figure. This construction keeps the

[1] Ann. Physik, 46, 357, 1915.

[2] Phys. Rev., 8, 48, July, 1916.

[3] Gen. Elect. Rev., 19, 1060, Dec., 1916.

which is Einstein's photoelectric equation. k must be determined by experiment; the measurements of Millikan give strong support to the correctness of this equation. And further, his results show that k is identical with Planck's radiation constant. From Tolman's point of view this is a very interesting result, but from the standpoint of Bridgman it is a necessary condition that the application of Tolman's equations give a correct result.

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Laboratory of Physics,
University of Illinois,
January, 1916.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. IX., No. 3, March, 1917.]

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BY CHAS. T. KNIPP.

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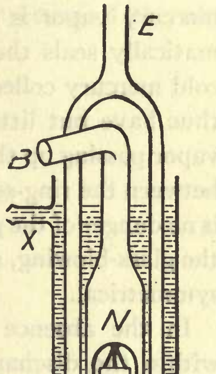

Fig. 1.
An improved high vacuum mercury vapor pump.

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[2] PHYS. REV., 8, 48, July, 1916.
[3] Gen. Elect. Rev., 19, 1060, Dec., 1916.

mercury, which collects at the ring-seal 3, cool, and thus removes the objection that mercury vapor having an *upward* velocity would enter the annular condensing chamber. A small opening shown at 3 serves as a valve which allows the accumulated mercury to pass, yet due to surface tension maintains a perfect seal. The short tube *P* is inserted to shield the hot mercury vapor streaming up from the boiler from condensing on the surfaces at 3. The upper end of *P* telescopes loosely into the lower end of *O*, while the lower end is secured by the ring-seal 1, having also a small valve opening in it through which the mercury passes back into the boiler. By making the upper end of *P* conical condensed mercury vapor is caught in the annular space thus formed and automatically seals the space *PO* from the cavity just outside of *P*. The cold mercury collected at the ring-seal 3, and the adjacent water-jacket, thus have but little opportunity of cooling the hot stream of mercury vapor passing up through *PO*, and, furthermore, the temperature gradient between the ring-seals 1 and 2, and 3 and *b b* are not abrupt, hence there is no danger of the glass cracking. This construction very much simplifies the glass-blowing, since the tube throughout the process is kept perfectly symmetrical.

In the absence of a convenient means of measuring the pressure within the discharge vessel quickly, the writer has chosen to express the degree of exhaustion in terms of the cathode dark space and the sparking distance at a parallel gap in air. The induction coil used was a 10 cm. spark Kohl coil, and the parallel gap was between balls 1.75 cm. in diameter. The supporting pump was a Gaede rotary mercury pump backed by a Gaede oil box pump. The mercury was heated in an oil bath by one bunsen burner. The trap beyond *B* was immersed in liquid air. Mercury vapor pumps are not sensibly operative until the vacuum drawn by the supporting pump is of the order of 1 cm. Crookes dark space, depending upon the construction of the pump as regards annular throat, etc. Hence the test for speed should not be from atmospheric pressure. The following tests serve to give an indication of the speed attainable.

Test 1.—With the discharge vessel fused to *B* (through a centimeter tube 30 cm. long) having a volume of 270 c.c. it required, by repeated trial, 29 minutes to lengthen the dark space from 1 cm. to 5 cm. equivalent spark in air; while with *the mercury vapor pump operative it required but 43 seconds.*

Test 2.—With a 6 liter vessel attached by a short large-diameter tube, it required 51 minutes to range from 1 cm. dark space to an equivalent parallel gap in air of 4.65 cm.; which time interval with *this pump*

operative was reduced to 2½ minutes. At the end of another 2 minutes the vacuum was so hard that the 10 cm. Kohl coil was not able to force a discharge.

The advantages of this form of hot blast mercury vapor condensation glass pump may be briefly stated as

1. The symmetrical design simplifies the glass-blowing.
2. Full effectiveness of the hot blast of mercury vapor without sensible loss of heat through a long delivery tube.
3. Effective cooling by a proper placing of water-jacket, ring-seals, and of an internal shielding tube.
4. The use of simple mercury valves for the direct return of the condensed mercury vapor to the boiler.

Laboratory of Physics,
University of Illinois,
January 15, 1917.

Reprinted from Vol. XVII, No. 5, May, 1917, School Science and Mathematics.

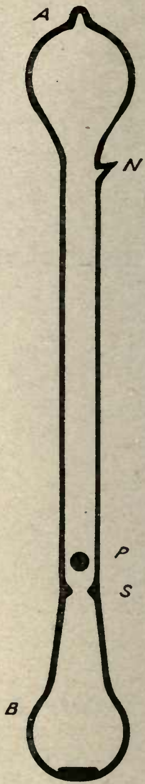
A SIMPLE DEMONSTRATION TUBE FOR EXHIBITING THE MERCURY HAMMER, GLOW BY MERCURY FRICTION, AND THE VAPORIZATION OF MERCURY AT REDUCED PRESSURE.[1]

By Charles T. Knipp,
University of Illinois, Urbana.

When the pressure over mercury is reduced to that of mercury vapor only, vaporization with heat takes place at surprisingly low temperatures, and the resulting mechanical pressure exerted by the issuing vapor from the mercury surface is even more surprising. The magnitude of this pressure over a surface confined in a large bulb, so that the vapor stream is not concentrated, is sufficient even at temperatures as low as 130° C. to freely support bits of cork. To make this easy of demonstration the writer has designed a tube to show the above, together with the familiar mercury hammer, and glow by mercury friction phenomena—all in one.

[1]Apparatus exhibited and demonstrated before the Galesburg meeting of the Illinois Academy of Science, February 24, 1917.

The tube should be about 35 cm. long by 1½ cm. in diameter, and have the usual bulb at each end that obtains for the mercury hammer. A stricture reducing the diameter to ½ cm. is placed near one end. A small quantity of mercury (about 10 grams) is put in the tube, and also a spherical pith ball about ¾ cm. in diameter is placed in the bulb farthest removed from the stricture. The tube is pumped out carefully and sealed off (the sealing-off nipple should be attached to the stem and not to one of the bulbs). It is now ready for the exhibition of the three phenomena referred to above. To show the pressure of the mercury vapor it is only necessary to hold the tube by the upper bulb (the one farthest from the stricture) over a Bunsen burner and allow it to heat gently. Soon condensed mercury vapor appears on the walls of the lower bulb and its progress up the tube is readily followed. The *bombardment of the mercury vapor lifts the pith ball* which, oscillating up and down, is forced into the upper bulb where it is violently agitated by the expanding mercury vapor stream. Removing the apparatus from the heat allows the oscillating pith ball to descend the tube until it again rests upon the stricture. At this moment if the bulb is returned to the flame the ball is again almost instantly shot upward—showing in a striking manner how sensitive the apparatus is. There is little danger of cracking the tube if care is taken not to plunge the lower bulb suddenly into the flame, and if it is allowed to cool before laying down.



The other two phenomena, i. e., the mercury hammer, and glow by mercury friction, are so familiar that they do not call for special mention here.

DISTRIBUTION OF POTENTIAL IN A CORONA TUBE.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. X, No. 3, September, 1917.]

DISTRIBUTION OF POTENTIAL IN A CORONA TUBE.

BY HARRY T. BOOTH.

I. INTRODUCTION.

1. *General Characteristics of D.-C. Corona.*—The name corona has been applied collectively to the conduction phenomena appearing when a sufficiently high potential difference is applied to two electrodes (two parallel wires, or two coaxial cylinders) separated by a gas. Corona appears for both alternating and direct impressed potential differences; for the purpose of our investigation, however, direct current corona was the more suitable.

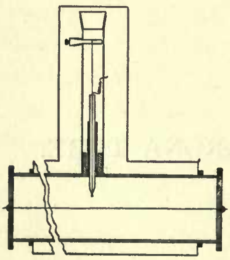


Fig. a.

Since a knowledge of the distribution of potential between the electrodes will be necessary for any fundamental corona theory, an investigation has been carried out at this laboratory to determine the field at every point between a wire and a coaxial tube, under various conditions of impressed voltage, pressure, size of wire, and current. It is hoped that the data taken will aid in the formulation of an adequate corona theory.

II. METHOD.

The distribution of potential between a wire and a coaxial cylinder was investigated in the following manner.

A hole was drilled in the side of a cylinder, and an insulated wire terminating in a bare spherical tip was arranged so that it could be moved radially between the wire and the tube. A micrometer microscope directed on a fixed point of the movable wire served to determine the relative position of the point. An electrostatic voltmeter of small capacity was connected in series with the exploring point and the tube.

When the point was moved to any portion of the radial field, the voltmeter quickly showed a constant deflection, indicating that the potential of the point was in equilibrium with that of the field at that particular place.

By moving the exploring point from the tube to the wire, observing the voltmeter readings at certain intervals, a comparatively accurate estimate of the intensity of the field was obtained.

III. Apparatus.

1. *The Corona Tube.*—The corona tube as indicated in the accompanying sketch was 35.5 cm. long and 7 cm. in diameter. The central wire was of copper, well polished, and stretched tightly. In all, four wires were used, No. 40, No. 32, No. 28 and No. 20 B. & S. gauge.

The ends of the tube were covered with heavy plate glass, drilled for the central wire, and sealed fast with half and half wax.

Since it was necessary to work at pressures lower than atmospheric, a glass tube was sealed over the exploring rod, so arranged with ground joints and springs as to allow the point to be moved at will without destroying the constant pressure.

2. *Source of Potential.*—The source of continuous potentials used in this set of investigations consisted of a battery of 40 500 volt, 0.5 ampere, shunt-wound, D.-C. generators connected in series.

These were arranged so that the potential could be varied continuously from about 300 volts up to 20,000 volts. Power for the driving motors was supplied by a motor generator set equipped with a voltage regulator, so that the voltage variation on the 110-volt power line was constant to within less than .5 per cent.

In general, the potential of the high tension line was as constant as the accuracy of the work demanded.

3. *Voltmeters.*—For the measurement of voltages, three voltmeters were used, a Kelvin electrostatic voltmeter with three ranges, a Braun electrostatic voltmeter, and a General Electric electrometer type voltmeter.

These instruments were calibrated with an attracted disc electrometer, equipped with a scale and vernier so that the distance between plates could be read to 0.05 mm. The force on the disc was measured by a fine balance.

The Braun voltmeter had a range of 0–3,500 volts, and since it is essentially an electroscope, it was almost ideal for use with an exploring point.

The Kelvin instrument had 3 ranges, 0–5000, 2,000–10,000, and 4,000–20,000 volts.

4. *Current Measurements.*—Currents between the wire and the tube were measured by means of a D'Arsonval galvanometer, used in connection with an Ayrton universal shunt. The figure of merit of the galvanometer was obtained, using standard resistances and a dry cell whose E.M.F. had been determined by comparison with a standard cell.

TABLE OF CURVES.

Figure.	Curve.	Wire B & S Gauge.	Voltage.	*I* Amperes.	*P* Mm. of Hg.	Temp. °C.	Remarks.
1	1	20	12,500	$9.76.10^{-6}$	745	25°	Faint glow
	2	20	13,850	$6.62.10^{-5}$	745	25°	Good glow
	3	20	15,420	$1.6\ .10^{-4}$	745	25°	Good glow
	4	20	16,000	$1.78.10^{-4}$	745	25°	Good glow
2	1	20	1,450	$3.9\ .10^{-5}$	23.5	27°	Dull glow
	2	20	2,150	$2.31.10^{-4}$	23.5	27°	Bright glow
	3	20	2,950	$5.58.10^{-4}$	23.5	27°	Brilliant purple glow
	4	20	2,150	Electrostatic curve			
3	1	20	10,000	$9.23.10^{-5}$	450	27°	3 or 4 steady beads wire negative
4	1	28	8,400	$3.19.10^{-6}$	745	25°	No apparent glow
	2	28	10,200	$2.66.10^{-5}$	745	25°	Faint glow
	3	28	11,500	$7.1\ .10^{-5}$	745	25°	Dull glow
	4	28	13,450	$1.95.10^{-4}$	745	25°	Good glow
	5	28	14,000	$3.73.10^{-4}$	745	25°	Bright glow
5	1	28	1,520	$4.43.10^{-5}$	19	24°	Good glow
	2	28	1,750	$1.35.10^{-4}$	19	24°	Good glow
	3	28	2,320	$3.73.10^{-4}$	19	24°	Bright glow
	4	28	2,890	$6.92.10^{-4}$	19	24°	Brilliant glow
	5	28	2,320	Electrostatic curve			
6	1	28	1,800	$9.48.10^{-4}$	19	24°	About 30 steady beads
7	1	32	6,510	$4.17.10^{-8}$	747	25°	No glow
	2	32	6,825	$1.91.10^{-6}$	747	26°	Distinct glow
	3	32	7,425	$1.91\ 10^{-5}$	747	26°	Good glow
	4	32	8,400	$5.94.10^{-5}$	747	26°	Good glow
	5	32	9,900	$9.54.10^{-5}$	747	26°	Bright glow
8	1	32	6,825	$1.91.10^{-6}$	747	26°	Distinct glow
	2	32	6,825	$2.03.10^{-4}$	241	24°	Bright glow
	3	32	6,825	$3.46.10^{-4}$	885	24°	Brilliant glow
	4	32	6,825	Electrostatic curve			
9	1	32	5,050	$1.79.10^{-8}$	744	26°	No glow
	2	32	5,650	$2.39.10^{-6}$	744	26°	A few dull beads
	3	32	7,250	$3.10.10^{-5}$	744	26°	Beads 1 cm. apart
10	1	40	4,520	$4.77.10^{-8}$	740	22°	No glow
	2	40	4,700	$1.19.10^{-6}$	740	22°	Distinct glow
	3	40	6,500	$2.26.10^{-5}$	740	22°	Good glow
	4	40	8,400	$8.29.10^{-5}$	740	22°	Good glow
	5	40	9,900	$1.67.10^{-4}$	740	22°	Brilliant glow
	6	40	8,400	Electrostatic curve			

IV. RESULTS.

1. *General Type of Curves.*—By the method of exploration already described, curves for the distribution of potential between wire and tube were taken for No. 40, No. 32, No. 28 and No. 20 copper wires stretched along the axes of the tube. These curves were taken for various pressures and voltages after the appearance of the corona. Representative curves obtained are shown in Figs. 1 to 10, and the conditions under which each curve was taken are given in Table I.

For the No. 40 wire, it was found impossible to obtain curves of the

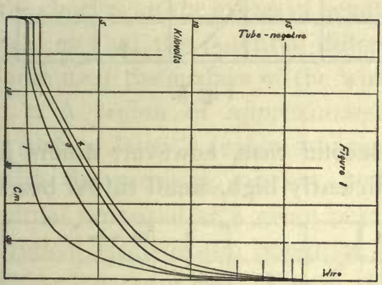


Fig. 1.

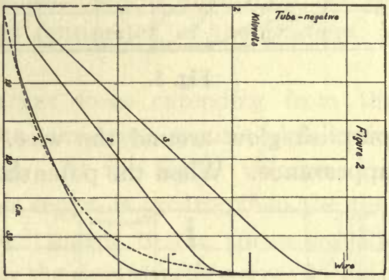


Fig. 2.

potential distribution when the wire was negative; for a given position of the exploring point the readings of the voltmeter were not constant. The beads appearing when the wire is negative were seldom at rest, and this would lead to the conclusion that each movement of the beads is accompanied by a change in the field surrounding the wire.

For No. 32 wire, when the wire was negative, two curves shown in Fig. 9

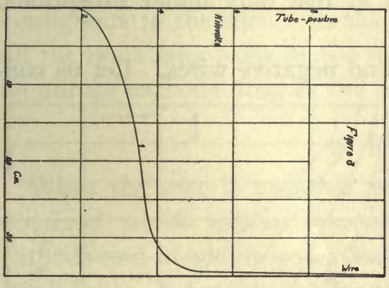


Fig. 3.

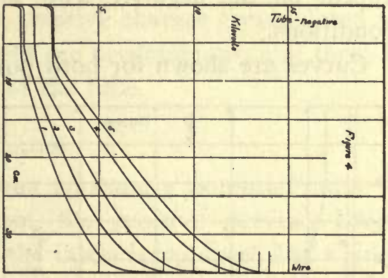


Fig. 4.

were taken before the corona appeared, also a portion of a curve for a voltage at which there was a distinct series of beads along the wire.

Curves were also obtained for No. 28 and No. 20 wire when the wires were negative, the same general characteristics being exhibited in each.

3. *Discussion of Curves.*—The corona discharge in general is divided

into two classes, *according as* (1) the wire is positive, and (2) the wire is negative.

The first case, when the wire is positive, is characterized by a uniform

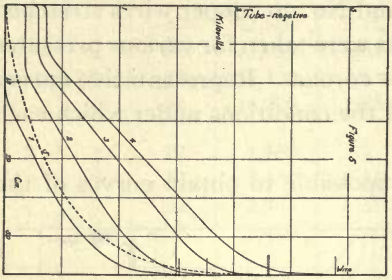


Fig. 5.

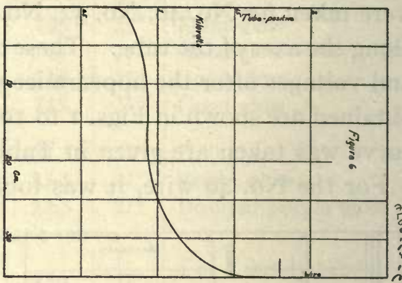


Fig. 6.

purplish glow around the wire. The second case, however, differs in appearance. When the potential is sufficiently high, small tufted beads

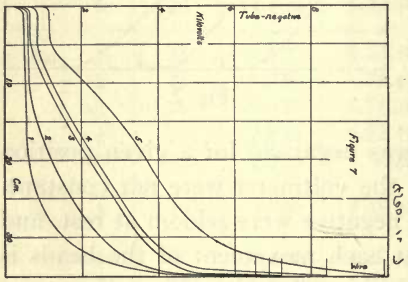


Fig. 7.

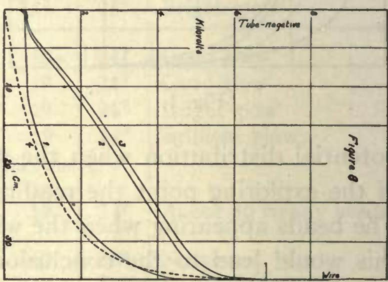


Fig. 8.

appear on the negative wire, and are at rest only under exceptional conditions.

Curves are shown for both positive and negative wires. Let us con-

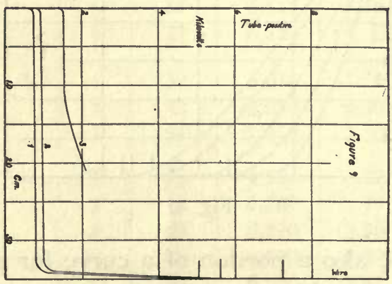


Fig. 9.

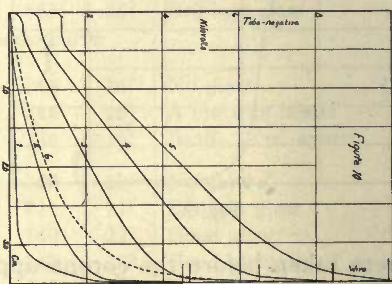


Fig. 10.

sider the appearance of the potential distribution curves when the wire is positive.

1. *The Positive Wire.*

In general, the space between the anode and the cathode may be broken up into four regions.

1. A region immediately surrounding the wire, which is characterized by a very large potential gradient. This may be due to the excess of the number of ions or electrons approaching the electrode over the number of those leaving, since the former number includes ions generated at all parts of the field, whereas the latter contain only ions that are generated in the narrow layer close to the wire. Thus we can see that the charges on the excess of negative ions near the wire disturb the electric field so that the potential difference per centimeter, or the gradient, is large near the surface of the wire.

2. A region of approximately constant force extending from the "surface layer" region adjacent to the wire, to a point which varies with the pressure, current, and voltage. At the higher voltages, the actual potential at a given point in this region is greater than the theoretical electrostatic potential, and the tangent to the curve may be either greater or less. Figs. 2 and 5 show the electrostatic curve (dotted), in comparison with actual curves taken.

3. A region of little or no force near the tube. In passing from II. to III. the number of positive ions increases (since they are generated in all the space between the wire and region III.), and their charges oppose those on the negative ions to such a degree that not only the negative charges on the ions, but also the electrostatic forces due to the configuration of the system are neutralized.

4. A region close to the tube, corresponding to the "surface layer" contiguous to the wire. In this space, positive charges accumulated at all the remaining parts of the radial field are predominant, and three is an abrupt cathode drop at the surface of the tube.

2. *Wire Negative.*

When the wire is negative and corona appears, a potential curve is obtained which differs somewhat from the positive curves. Large cathode and anode drops appear, and the intervening space has a very small field. Reasoning similar to that explaining the shape of the curves when the wire is positive explains the negative curves.

So in general, the anode and cathode drops of potential are predominant in both types of curves. There are several reasons for this, namely:

1. Polarization potential between a metal and a gas.
2. Accumulation of ions.
3. Reflection of ions.

4. Different velocities of positive and negative ions.
5. A non-uniform field.

The Potential Curves from a Theoretical Point of View.

1. The starting point of the corona.

We have Peeks empirical formula for the starting intensity,

$$E_1 = E_0\left(1 + \frac{\beta}{\sqrt{R_1'}}\right), \tag{1}$$

where E_1 is the force at the surface of the wire of radius R_1 and E_0 and β are constants.

From the general electrostatic theory, at the moment when the corona discharge is starting, just before the field has been disturbed by the moving charges,

$$E_1 = \frac{(V_1 - V_2)}{R_1 \log \dfrac{R_2}{R_1}}. \tag{2}$$

Therefore at the instant when the corona starts

$$E_0\left(1 + \frac{\beta}{\sqrt{R_1}}\right) = \frac{(V_1 - V_2)}{R_1 \log \dfrac{R_2}{R_1}} \tag{3}$$

or

$$E_0 = \frac{(V_1 - V_2)}{R_1 + \beta\sqrt{R_1}} \frac{1}{R_1 \log \dfrac{R_2}{R_1}}, \tag{4}$$

which resembles the general formula for the electric force between two concentric cylinders,

$$E = \frac{(V_1 - V_2)}{r \log \dfrac{R_2}{R_1}}. \tag{5}$$

Hence, when $r = R_1 + \beta\sqrt{R_1}$,

$$E = E_0.$$

2. Calculation of the volume density of electrification in the space between the two concentric cylinders.

For a system where the potential at a point is due to moving charges as well as static charges, we have Poisson's equation expressing the density in terms of the potential,

$$\nabla^2 V = -4\pi\rho, \tag{6}$$

or, writing it in cylindrical coördinates,

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2} = -4\pi\rho. \tag{7}$$

For this particular case, the derivatives in z and Φ are zero, so rewriting the above equation, using total derivatives,

$$\frac{d^2V}{dr^2} + \frac{1}{r}\frac{dV}{dr} = -4\pi\rho. \tag{8}$$

Since the density is an undetermined function of the radius, the equation cannot be integrated directly. If, however, we plot the potential against the distance from the axis, a graphical method will aid in the determination of the density. That is, if the first derivative of the potential is determined from the curve for a series of values of r, these new values may be plotted against the radius again. By repeating this process with the derived curve, a relation between the second space derivative and the radius is obtained. From these two derived curves, then, the density may be computed according to equation (8).

Fig. 11 is a repetition of Curve 4, Fig. 1, and Fig. 12 shows the density as computed for the different values of r.

The density curve shows what we have deduced intuitively in regard

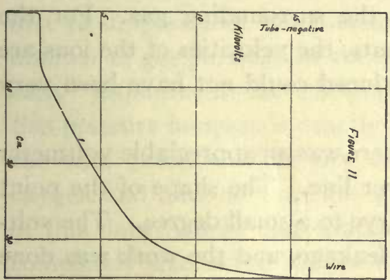


Fig. 11.

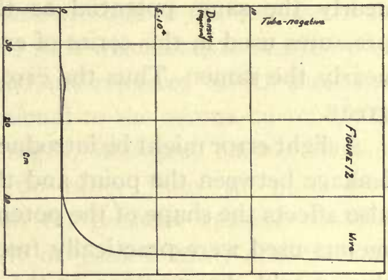


Fig. 12.

to the charges necessary to produce the observed distortion of the field. The large resultant negative charge near the positive wire and the positive charge near the negative tube should be expected. A peculiar maximum appears at about 2.7 cm. from the wire (Fig. 12).

4. *Sources of Error.*

1. Potential assumed by a sphere in an ionized gas.

It is difficult to draw conclusions as to the absolute potential of a sphere in a conducting gas, since it is very likely that the potential at an undisturbed point in a gas is not the same as the potential assumed by a sphere when its center is at this point.

In the case of a sphere near the positive electrode, its potential being initially the same as that of the gas, two streams of ions move in opposite directions past the side of the sphere, one containing a large number of

negative ions, and the other a smaller number of positive ions. It intercepts more negative ions than positive, so that its potential falls below that of the surrounding gas. The charge thus acquired by the sphere increases until the effect which it produces in attracting positive and repelling negative ions causes them to come in contact with the sphere in equal numbers. The final value of the potential assumed by the sphere is too high by an amount which depends upon the relative velocities of the positive and negative ions.

Conversely, when the exploring sphere is close to the negative electrode, there are a greater number of positive ions intercepted than negative ions, so that the potential of the sphere rises above the potential of the undisturbed gas, until finally an equilibrium is reached, the number of positive charges acquired by the sphere being equal to the number of negative charges. Thus the potential assumed by the sphere is greater than that of the undisturbed gas.

If, however, the velocity of the positive ions is approximately equal to that of the negative ions, then the exploring point should attain very nearly the same potential as that of the surrounding gas. For the pressures used in this series of experiments, the velocities of the ions are nearly the same. Thus the error introduced could not have been very great.

A slight error might be introduced if there was an appreciable voltmeter leakage between the point and the power line. The shape of the point also affects the shape of the potential curve to a small degree. The voltmeters used were practically free from leakage, and the work was done during cold, dry weather, so the error introduced from this cause is negligible.

An attempt is being made to formulate the mathematical theory of the corona discharge, and it is hoped that these potential curves will aid in the solution of the problem.

Summary.—The distribution of potential between the electrodes of a corona tube was determined for four sizes of wire, for various pressures and potential differences. From these curves the density of the charge along the radius was derived by means of graphical methods.

In conclusion, I wish to express my appreciation of the suggestions and advice given by Dr. Jakob Kunz, of this laboratory, and to Mr. J. W. Davis and Mr. R. W. Owens for the use of portions of their data on this problem.

PHYSICS LABORATORY,
UNIVERSITY OF ILLINOIS,
May 11, 1917.

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THE PRESSURE INCREASE IN THE CORONA.

BY EARLE H. WARNER.

I. INTRODUCTION.

IT has been reported by Farwell and Kunz that at the instant the corona appears about an axial wire in a cylindrical tube, the pressure of the gas in the tube suddenly increases.[1] It has always been stated that this pressure increase could not be due to heat, because of the instantaneous character of its appearance, and because of the rapidity with which it disappears as soon as the potential is removed from the wire. Since the only theories which have been advanced to explain the corona assume it to be an ionization phenomenon, it seemed reasonable to suppose that this pressure increase was due to the increase in the number of gas particles in the tube, and so it was called ionization pressure. Experiments have been performed and reported[2] which show that this pressure increase is exactly proportional to the corona current, with the wire positive when dry air, hydrogen, nitrogen, carbon dioxide, oxygen and ammonia are the gases in the tube. Since the publication of this data Arnold[3] has contended that the pressure increase could be completely accounted for as the result of Joule's heat, and that the assumption that it is due to ionization is untenable. To support this contention Arnold performed experiments " by electrically heating the central wire in apparatus similar to Farwell's and " observed the pressure increase. With such an apparatus Arnold attempted to show (1) that an increase in pressure due to heat appears suddenly, (2) that for a given power consumed in the tube the increase in pressure due to heat is of about " the same magnitude as those observed " in the corona.

In order to show clearly that the pressure increase is not due to heat a series of comparative experiments were performed with the pressure increase caused, first, by producing the corona glow on the wire and, second, by heating the central wire. The pressure increase observed in the first set of experiments will be referred to as *caused by corona* and in the second set as *caused by heat*.

[1] Dr. S. P. Farwell, "The Corona Produced by Continuous Potentials," Proc. A. I. E. E. Nov., 1914. Dr. Jakob Kunz, "On the Initial Condition of the Corona Discharge," PHYS. REV., July, 1916.

[2] Earle H. Warner, "Determination of the Laws Relating Ionization Pressure to the Current in the Corona of Constant Potentials," PHYS. REV., Sept., 1916.

[3] H. D. Arnold, (Abstract) PHYS. REV., Jan., 1917.

A few computations have also been made which strengthen the results of the experiments.

II. Experimental Results.

1. The reason why one who sees this pressure increase, as recorded by a quick-acting pressure meter, thinks it is not a heat effect, is because of rapidity with which it appears and disappears. Arnold showed that the pressure increase occurred quite rapidly when caused by heat. The following curves show the difference in the rapidity of appearance and disappearance of the pressure increase caused by heat, and caused by corona. It will be noticed in Fig. 1, where the pressure increase was caused by heating the central wire, that fifteen seconds was required for the prssure to come to its maximum value, and that from the time the current was broken twenty-five seconds was required for the pressure to return to practically its original value, while in Fig. 2, where the pressure increase was caused by corona, only three seconds was required for the maximum pressure to be attained and that the pressure came back to practically its original value in eighteen seconds. In this last case from the appearance of the phenomenon it seems, if the aneroid pressure meter had less inertia, that the pressure increase could be determined in less than three seconds. These curves show that the pressure increase appears five times as rapidly when caused by corona as when caused by heat, and disappears also more rapidly.

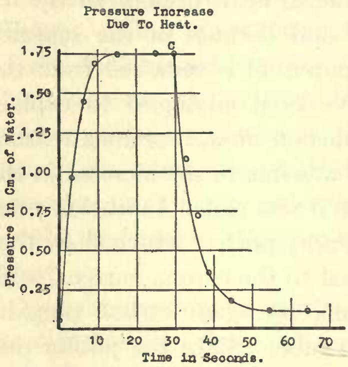


Fig. 1.

2. In the pressure increase due to corona, a short time interval of five to seven seconds occurs after the sudden increase of pressure, before the heat effect in the corona begins to be noticed. This is shown by an abrupt bend, A, in the curve where the pressure increase is plotted against time, as is done in Fig. 3. No such bend occurs in the case where the pressure increase is caused by heat alone, as is shown in Fig. 1. In the work which has previously been reported the pressure increase measurements were always taken at the point A, and this seems to be practically independent of the heat effect.

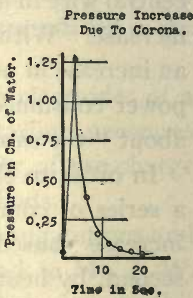


Fig. 2.

3. The heat which is produced in the corona discharge, shown by the gradual pressure increase from B to C, Fig. 3, is distributed throughout the whole volume of enclosed air and so, when the current is broken does not radiate rapidly because the air is a poor conductor. This is shown very clearly in Fig. 4. This seems to show that the pressure increase due to

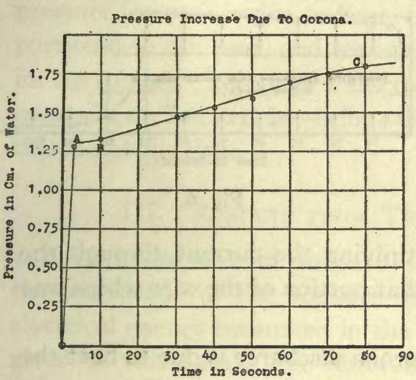


Fig. 3.

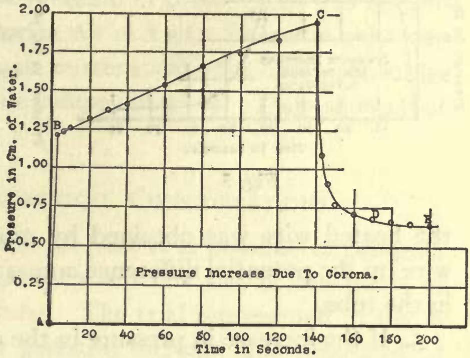


Fig. 4.

heat in the corona is represented by the difference of ordinates of C and B (Fig. 4). As soon as the corona current is broken at C the increase in pressure due to corona at once disappears, but the increase in pressure due to heat in the corona discharge remains, as is shown by the difference of ordinates of D and A. This difference is always very nearly equal to the difference of ordinates of C and B. This heat energy produced by the corona current, since it is distributed through the gas, radiates very slowly, as is shown by the gradual descent of the curve from D to E. No such effect is observed when the increase of pressure is due entirely to heat, as is shown in Fig. 1. This curve (Fig. 1) shows that twenty-five seconds after the current through the wire is broken at C the resultant pressure increase due to heat has practically disappeared; while Fig. 4 shows that twenty-five seconds after the corona is removed from the wire the increase in pressure due to the corona has disappeared, but practically all the pressure increase due to heat in the corona (ordinates C minus B approximately equals ordinates D minus A) still remains and radiates very slowly.

4. If the increase in pressure is due to heat, the same increase in pressure should result when the same power is consumed (a) with a corona current through the gas, (b) with a heating current through the wire. Figs. 5 and 6 show that this is not the case. The powers consumed in the two cases are not exactly the same, but one can see that were they the same, the increase in pressure due to corona would be approxi-

mately one half the increase in pressure due to heat. The power in the case of the corona was obtained by multiplying the potential difference between the wire and the tube by the corona current, and in the case of

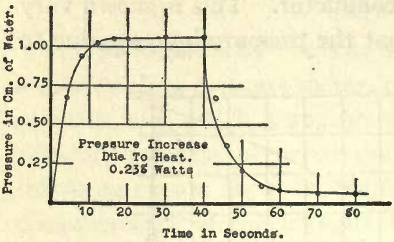


Fig. 5.

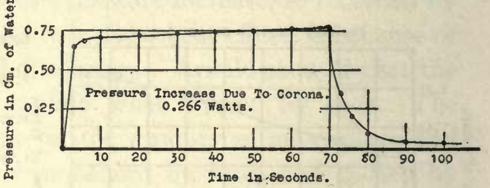


Fig. 6.

the heated wire was obtained by multiplying the current through the wire by the potential difference across that portion of the wire which was in the tube.

5. If the increase in pressure in the corona discharge is due to heat the temperature of the air in the corona tube must increase. This may or may not be the case in the luminous layer near the wire but the temperature of the gas in the tube at a point four millimeters from the wire actually decreases. This was determined by inserting a sensitive thermocouple made of very fine Copper-Advance wire into the corona tube. The temperature decreased only at the instant the corona appeared. In a short time, after the heat due to the corona began to appear (corresponding to the slope *B* to *C*, Figs. 3 and 4) the temperature of the gas in the tube began to increase. This cooling effect is shown in Fig. 7. Comparing Figs. 7 and 3 it is seen that the increase in pressure which was measured at *A* was observed while there was an actual cooling in the corona tube. This cooling should be expected when air or oxygen are in the tube, for under these conditions ozone is formed. Since the formation of ozone from oxygen is always accompanied with an absorption of heat the temperature of the air or oxygen would tend to lower. Mr. J. W. Davis, working on corona about hot wires in hydrogen, has discovered that the appearance of the corona about a tungsten wire heated to white heat, causes it to cool to dull red. This tends to show that even in the corona glow itself there is a cooling effect.

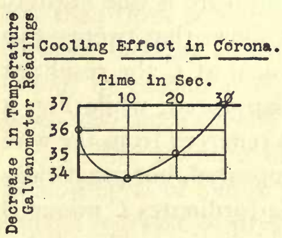


Fig. 7.

6. If the increase in pressure in the corona is due to heat one should expect it to be the same with the wire either positive or negative. As has been previously mentioned it is impossible to obtain measurements

when the wire is negative because of the presence of beads. The negative corona is entirely different from the positive corona.

7. The following consideration will further show that the increase in pressure can not be due to heat. The heat produced by the corona current will be given by the equation $H = 0.238\, eit$ and, if the observed pressure increase is due to heat, the increase in pressure Δp will be proportional to the heat, and we can write $\Delta p = k\, eit$. Now the only way for Δp to vary directly as i, the corona current, as is the case—shown by curves in the last article—is for e to be independent of i. Data shows that this is not the case.

III. Results from Theoretical Considerations.

1. If the increase in pressure is due to heat it is possible to compute the magnitude of the pressure increase when one knows the watts of electrical energy consumed in the tube. The trial represented in Fig. 6 gives us this data. The observed pressure increase was measured in three seconds so that the total number of joules of work consumed by the tube in that time was $3 \times 0.266 = 0.798$ joules and this corresponds to 0.1909 calories. Knowing the volume of the tube, the temperature and pressure of the air in it, the mass of the air in the tube can be computed. With the above-mentioned quantity of heat and mass of air, together with the specific heat of the air at constant volume, the temperature rise of the air can be computed, assuming that the electrical energy is converted into heat. This temperature rise comes out to be 2.44° C., which at constant volume corresponds to a pressure increase of about nine cm. of water, while the observed pressure increase in this particular trial amounts to about seven tenths cm. of water. In this computation radiation and conduction losses have been neglected because they would be very small from a body 2.44° C. above room temperature. This shows that the observed results lie in a different order of magnitude from what would be expected if Arnold's theory were true.

2. Arnold states, if "we compute the corona currents that would result from the presence of enough ionized particles to produce the observed pressure changes, the currents calculated are many thousand times greater than those actually obtained." Such a statement is only true when the ionized particles are produced in a uniform or practically uniform electric field. This is not the case in the corona tube. H. T. Booth is publishing data on the distortion of the field in the corona tube. This data shows that the potential gradient near the wire is very high—of the order of 30,000 volts per cm. This is the arcing gradient, in which it is probable every molecule is ionized. Then for a long space between the

wire and the tube there is a very small gradient. With this condition of the field, near the wire every molecule may be ionized and still the resultant current be very small, for few of the ionized particles near the wire will pass through the space where there is a small gradient. Simple computations based on kinetic theory show that the maximum observed pressure increases can be explained by ionization if every molecule of the air within 1.39 mm. of the wire is ionized. Within this distance the potential gradient is equal to the arcing gradient and therefore probable that all molecules are ionized.

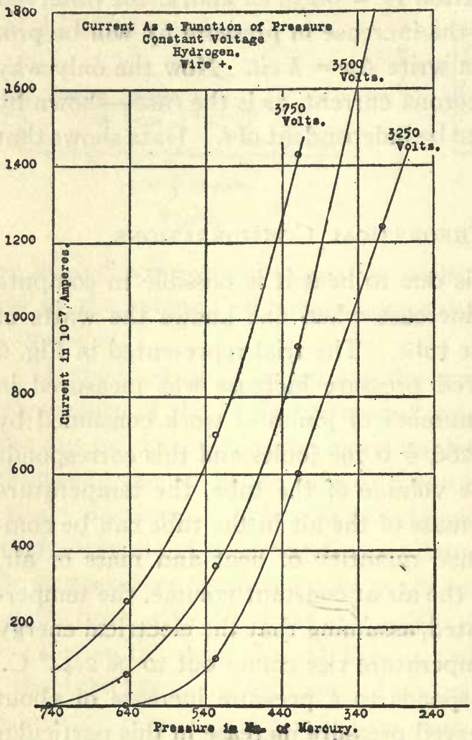


Fig. 8.

IV. Further Verification of Kunz's Theory.

The final equation as presented in the last article is

$$ki = \frac{v_0}{e}(p_1 - p_0) + \text{a constant},$$

where i is the corona current, v_0 the volume of the tube, e the potential difference between the wire and the tube, $p_1 - p_0$ the pressure increase, k a constant and p_0 the initial pressure. This equation shows that for a constant potential difference e, the current i should increase as p_0 is lowered. Data were taken, by measuring the current at various measured pressures, caused by a constant potential difference, which verifies this theory. These data are shown graphically in Figs. 8 and 9 when pure hydrogen and nitrogen respectively were the gases in the tube.

V. Summary and Conclusions.

Experimental results show:

1. That the increase in pressure due to corona appears and disappears much more rapidly than when due simply to heat.

2. That the heat in the corona discharge is not a prominent factor until many seconds after the corona appears.

3. That in equal energy experiments the increase in pressure due to corona differs from the increase in pressure due to heat by about 50 per cent.

4. That at the instant the corona appears the gas in the tube at a small distance from the wire is cooled.

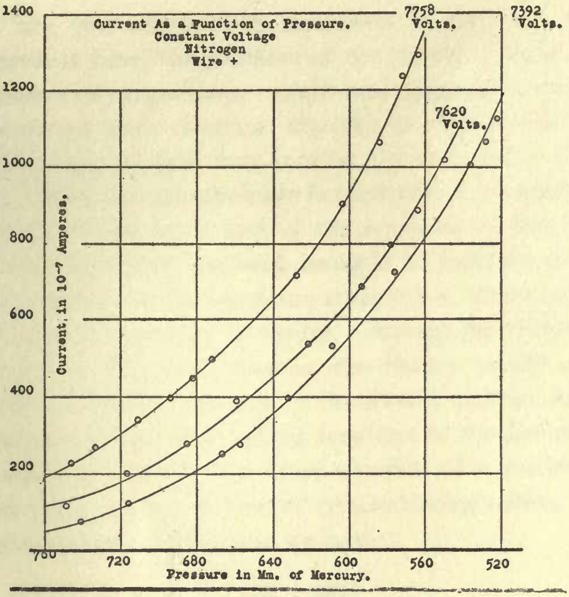


Fig. 9.

5. That the theory advanced by Kunz is verified in one more field, namely in the relation between current and pressure for constant voltage.

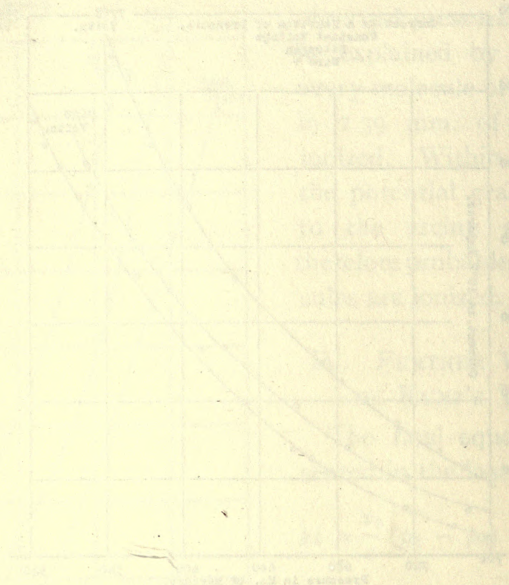
These results together with conclusions drawn from simple calculations, force one to believe that the pressure increase in the corona discharge is not due to Joule's heat. With the recent knowledge of the distortion of the field in the corona tube it seems very possible that the increase in pressure is due to ionization.

The writer desires to express his appreciation to Professor A. P. Carman for the use of the laboratory facilities, and to Dr. Jakob Kunz for his continued interest and suggestions.

Laboratory of Physics,
University of Illinois,
June, 1917.

3. That in equal energy experiments the increase in pressure due to corona differs from the increase in pressure due to heat by about 40 per cent.

4. That at the instant the corona appears the gas in the tube at a small distance from the wire is ionized.



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ON BOHR'S ATOM AND MAGNETISM.

BY JAKOB KUNZ.

THERE are two outstanding problems in the field of physics at the present time, the problem of the nature of light and its origin and the problem of magnetism. Light and magnetism seem to be very directly connected with electrical charges in motion and the ultimate theory of the origin of light may involve the solution of the problem of magnetism. Several attempts have been made at an explanation of the radiation of the black body and of the emission of line spectra. The most surprising fact that has been brought to light by these investigations is the existence of the quantum constant h, which seems to belong to the fundamental constants of nature. Among the various attempts at an explanation of the series spectra, the theory of the atom by Bohr deserves special attention because it involves h and accounts with very surprising accuracy for the Rydberg constant of the Balmer and related series. According to Bohr the atom consists of a nucleus surrounded by electrons revolving in stationary non-radiating orbits, for which the laws of electrostatics hold so that we have:

$$\frac{ee_1}{a^2} = \frac{mv^2}{a}.$$

The stationary circles are determined by the postulate that the kinetic energy $\frac{1}{2}mv^2$ is proportional to the quantum of energy such that

$$E_k = \tfrac{1}{2}mv^2 = \tfrac{1}{2}zhn \quad (z = \text{integer})$$

$$mv2\pi na = zhn$$

$$mva = m2\pi na^2 = \frac{zh}{2\pi};$$

hence the constant h is proportional to the moment of momentum of the revolving electron. It is also proportional to the magnetic moment M of the electron in the stationary orbit, thus

$$M = \pi a^2 i = \pi a^2 en = zh\frac{e}{4\pi m}.$$

The non-radiating orbit of the electron seems to account at once for the constant magnetic properties of the elements and their compounds.

When the electron moves from an outer to an inner stationary orbit, it loses a quantum of energy, giving rise to a line in the spectrum;

$$nh = E_{z_2} - E_{z_1} = \frac{2\pi^2 me^2 e_1^2}{h^2}\left(\frac{1}{z_2^2} - \frac{1}{z_1^2}\right). \tag{1}$$

Two of the fundamental problems seem to be solved at the same time, the electron, when jumping from an outer to an inner stationary orbit giving rise to light and when moving undisturbed in a stationary orbit, producing the magnetic effects. Of course even if both parts of the theory were verified, the question would still remain, why such non-radiating orbits are possible, in other words, why Maxwell's theory of electromagnetic radiation does not hold within the atom. If this theory is invalid within the atom, then we might expect also that the theory of relativity does not hold in the same regions.

Before I proceed to the discussion of magnetism on the basis of Bohr's theory, I wish to call attention to an interesting conclusion with regard to the velocities of those electrons near the nucleus which give rise to the Roentgen spectra, the approximate law of which has been discovered by Moseley. The square root of the highest frqeuencies from the atoms of the chemical elements is proportional to the charge e_1 of the nucleus. This law follows from Bohr's theory. If we call the nuclear charges of two atoms e_{11} and e_{12} and assume the factor $1/z_2 - 1/z_1^2$ to be the same in both elements, then we obtain from equation (1) for the highest frequencies of the two elements

$$\sqrt{\frac{n_1}{n_2}} = \frac{e_{11}}{e_{12}} = \frac{e_1}{e_2} = \frac{N_1}{N_2}$$

where N_1 and N_2 are the corresponding atomic numbers. Now, Millikan has tested this relation for tungsten and hydrogen and has concluded that the shortest wave-length which could be produced by hydrogen is 91.4, while Lyman found for the convergence wave-length 91.2 $\mu\mu$. This would correspond to the highest frequency of the hydrogen atom. It seems certain that the Lyman ultra-violet series of hydrogen lines is the K series of this element. While this agreement is very satisfactory, it should be remarked that we have as yet no proof of the existence of Balmer series in the Roentgen spectra, and the resolving power of the crystals for the characteristic Roentgen rays may be too small, so that it remains impossible with the present experimental means to discover series analogous to Balmer series in the Roentgen spectra. Now the relation

$$\sqrt{\frac{n_1}{n_2}} = \frac{e_1}{e_2} = \frac{N_1}{N_2}$$

may be true, even if Bohr's theory of the atom is not true, provided we introduce the quantum relation in the following way, as has been shown by F. Sanford.[1]

$$1)\ \tfrac{1}{2}mv^2 = hn \qquad v = 2\pi na = \sqrt{\frac{2h}{m}}\sqrt{n}$$

$$mva = h/\pi$$

$$2)\ mv^2/a = ee_1/a^2 \qquad e_1 = mv^2a/e = = vh/\pi e$$

$$e_1 = \sqrt{2/m}\frac{h^{3/2}}{\pi e}\sqrt{n_1}$$

$$\frac{e_1}{e_2} = \frac{n_1}{n_2}.$$

Whether we deduce this relation according to Sanford's or to Bohr's method we assume in both cases that the mass m of the electron is the same in both and in all elements.

In the theory of relativity the ratio of the transversal mass m of the moving electron to the mass m_0 of the electron at rest is given by

$$\frac{m}{m_0} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

If we assume 1.01 for m/m_0 we find $v = 4.2\cdot 10^9$ cm. per sec. If the shortest wave-length λ measured for tungsten is equal to $0.167\cdot 10^{-8}$ cm. then

$$n = \frac{c}{\lambda} = 1.8\cdot 10^{19}$$

$$\frac{n_1^2}{n_2^2} = \left(\frac{e_1}{e_2}\right)^4, \tag{3}$$

but

$$\frac{ee_1}{a^2} = \frac{mv^2}{a} = m4\pi^2n^2a$$

$$n^2 = \frac{ee_1}{m4\pi^2a^3}$$

$$\frac{n_1^2}{n_2^2} = \frac{e_{11}}{e_{12}}\left(\frac{a_2}{a_1}\right)^3 = \frac{e_1}{e_2}\left(\frac{a_2}{a_1}\right)^3 = \left(\frac{e_1}{e_2}\right)^4. \tag{4}$$

From (4)

$$\frac{e_1}{e_2} = \frac{a_2}{a_1}$$

[1] Physical Review, Vol. IX., p. 383, 1917.

for two different elements.

$$a_2 = a_1 \frac{e_1}{e_2} = a_1 \frac{N_1}{N_2}.$$

For the hydrogen atom

$$a_1 = 5.5 \cdot 10^{-9} \text{ cm.}$$

for tungsten

$$\frac{N_1}{N_2} = \frac{1}{74},$$

hence

$$a_2 = 7.43 \cdot 10^{-11} \text{ cm.}$$

for the innermost orbit of the electron in the tungsten atom, and the circular velocity

$$v_2 = 2\pi n a_2 = 8.38 \cdot 10^{+9} \text{ cm.}$$

For the uranium atom we would find a path velocity of the electron in the innermost orbit amounting to $2.1 \cdot 10^{10}$ cm., approaching somewhat the velocity of light. This velocity would correspond to a mass $m = 1.4 m_0$, if we neglect the influence of other revolving electrons. It is remarkable that these highest velocities of revolving electrons remain only about 30 per cent. below the velocity of light, on the other hand a satisfactory theory of the Roentgen spectra must take this effect into account or deny relativity in the interior of the atom.

The velocity of the electrons in the orbits of Bohr's atom is so great, that it seems possible to explain the magnetic properties of the elements by the assumption of a few electrons revolving in nonradiating orbits. We shall proceed to compare the magnetic properties of some of the simplest elements with Bohr's theory of the atoms and molecules. The first difficulty which we encounter consists in the fact that we have measured so far only the magnetic properties of molecules (except the rare gases) and not of atoms and that Bohr's theory of the molecules, of hydrogen for instance, is not so definite as that of the atoms. The hydrogen atom consists of a nucleus of charge e_1 and an electron of charge e, revolving in a circular orbit around about the nucleus. This atom represents an elementary magnet and if the hydrogen gas were made up of atoms it would be paramagnetic and the paramagnetic susceptibility could be evaluated at once. But now the question arises as to the nature of the coupling of two atoms in a molecule. The elementary magnets of two atoms may arrange themselves so that the axes of the two magnetons form the same line, hydrogen would then still be paramagnetic, or the axes may be parallel to each other and the neighboring poles be of opposite sign, the hydrogen gas would then be dia-

magnetic; finally, and this is the case Bohr has assumed for the hydrogen molecule, both electrons revolve in the same orbit, separated by 180°, the plane of the orbit being at right angles to the line joining the two nuclei. Bohr calculates for the radius of the common orbit of the electrons $a = 5.22 \cdot 10^{-9}$ cm., for the frequency $n = 6.72 \cdot 10^{15}$. This gas is paramagnetic. The magnetic moment M is equal to:

$$M = 2 \cdot \pi a^2 en = 1.82 \cdot 10^{-20}$$

The magnetic susceptibility at 0°

$$k = \frac{NM^2}{3RT} = \frac{2.72 \cdot 10^{19}(1.82)^2 \cdot 10^{-40}}{3 \cdot 1.\ 37 \cdot 10^{16} \cdot 273}$$

$$k = +\ 8.2 \cdot 10^{-8} \text{ per unit volume at } 0°.$$

The values which I find in the literature are contradictory: $+\ 0.8 \cdot 10^{-8}$ (Quincke), $-\ 0.5 \cdot 10^{-8}$ (Bernstein), $-\ 0.34 \cdot 10^{-8}$ (Blondlot). A further accurate determination of the magnetic properties of hydrogen is very necessary.

The helium atom in Bohr's theory consists of a nucleus of charge $2e$ around which there are rotating 2 electrons in the same orbit of radius $a = 0.314 \cdot 10^{-8}$ cm., with a frequency $n = 19.10^{15}$. This gas would be paramagnetic. The magnetic moment of the atom is equal to $1.85 \cdot 10^{-20}$, the susceptibility $k = 8.5 \cdot 10^{-8}$ at 0°; for both gases $k = C/T$ where Curie's constant $C = NM^2/3R$. Helium like the other inert gases is diamagnetic. Here Bohr's theory is in contradiction with the experimental fact.

Lithium is supposed to contain a nucleus of charge $3e$; two electrons revolve in the inner orbit and one electron in the outer orbit in the same direction, giving rise to paramagnetism; the magnetic moment can easily be calculated.

$$M = 2\pi a_1^2 en_1 + \pi a_2^2 en_2 = 2.815 \cdot 10^{-20},$$

$$a_1 = 1.99 \cdot 10^{-9} \qquad a_2 = 0.651 \cdot 10^{-8},$$

$$n_1 = 4.74 \cdot 10^{16} \qquad n_2 = 4.43 \cdot 10^{15}.$$

The paramagnetic susceptibility of lithium would be equal to:

$$k = \frac{NM^2}{3RT} = \frac{2.72 \cdot 10^{19}(2.815)^2 10^{-40}}{3 \cdot 1.37 \cdot 10^{-16}\ 273} = +\ 1.98 \cdot 10^{-7}$$

per unit volume at 0°. Lithium like the other alkali metals is weakly paramagnetic. The literature contains the value $2.26 \cdot 10^{-7}$, which agrees

with the theoretical value even better than we should expect, because we have treated lithium like a gas, while for the solid state the mutual action of the elementary magnets must be taken into account.

Finally, beryllium, in Bohr's theory, consists of a nuclear charge $4e$ with two orbits, each containing two electrons. If the radii and the frequencies of the electrons in the neutral state of the atom were uniquely determined, the magnetic moment and the magnetic susceptibility could easily be calculated. The atom model is paramagnetic in agreement with experimental determinations.

All four substances, hydrogen, helium, lithium, beryllium, are paramagnetic according to Bohr's theory, while hydrogen is probably diamagnetic and helium is almost certainly diamagnetic. The effect of a magnetic field on a paramagnetic gas consists in the orientation of the molecular magnets into the direction of the external field; so that there will be a state of equilibrium between the directing tendency of the field and the disturbing tendency of the temperature agitation. As far as this effect of the field is concerned, we are justified in applying the theory of paramagnetism to Bohr's atom model. But the field must also have a secondary effect on paramagnetism, an effect which determines at the same time the diamagnetic properties.

Let us consider in a diamagnetic gas an atom with an electronic orbit of radius a, the electron e revolving with velocity v in a plane perpendicular to the magnetic field. The magnetic moment, without the action of the external field, will be equal to $M = \pi a^2 en$; if a field H is applied the frequency will change so that

$$dM = \pi e(2anda + a^2dn)$$

or assuming the first term small,

$$dM = \pi ea^2dn = - \pi ea^2 \frac{dT}{T^2}.$$

But in the theory of diamagnetism as well as in Lorentz's theory of the simple Zeeman effect, it is assumed that

$$\frac{mv^2}{a} = f \cdot a,$$

that is, the centripetal force, which balances the centrifugal force is proportional to the distance a between the electron and the center of the atom, the centripetal force is a quasi elastic force; while in Bohr's theory the centripetal force is inversely proportional to the square of the distance between the electron and the nucleus. Yet even under these circumstances we can find, allowing certain approximations, the older

expressions for the diamagnetic susceptibility and for the Zeeman effect, except for a factor 2, as will be seen from the following deductions which are self explanatory.

$$\frac{mv^2}{a} = \frac{f}{a^2}$$

With a magnetic field we have:

$$\frac{mv^2}{a'} = \frac{f}{a'^2} - \mathrm{Hev} = \frac{mv^2a}{a'^2} - \mathrm{Hev}$$

$$\frac{mv^2}{a'^2} - \frac{mv^2a}{a'^3} = -\frac{\mathrm{Hev}}{a'}$$

$$v = \frac{2\pi a}{T} = \frac{2\pi a'}{T'},$$

$$\frac{v}{2\pi} = \frac{a}{T} = \frac{a'}{T'} = \text{Constant},$$

$$T' = \frac{a'}{a}T,$$

$$2\pi m\left(\frac{1}{T'^2} - \frac{1}{T'^2}\frac{a}{a'}\right) = -\frac{\mathrm{He}}{T'},$$

$$2\pi m\left(\frac{1}{T'^2} - \frac{a^3}{a'^3}\frac{1}{T^2}\right) = -\frac{\mathrm{He}}{T'},$$

$$\frac{2\pi m}{a'^3}\left(\frac{T^2a'^3 - T'^2a^3}{T^4}\right) = -\frac{\mathrm{He}}{T'},$$

but

$$a' = a + da, \quad a'^3 = a^3 + 3a^2da \quad T' = T + dT, \quad T'^2 = T^2 + 2TdT,$$

hence

$$\frac{2\pi m}{a'^3}\frac{T^2 3a^2da - a^3 2TdT}{T^4} = -\frac{\mathrm{He}}{T'},$$

$$da = \frac{v}{2\pi}dT = (2\pi a/2\pi T)dT = \frac{a}{T}dT,$$

$$\frac{2\pi m}{a'^3}\frac{3a^3TdT - 2a^3TdT}{T^4} = -\frac{\mathrm{He}}{T},$$

$$2\pi m\frac{a^3}{a'^3}\frac{dT}{T^2} = -\mathrm{He},$$

$$\frac{dT}{T^2} = -dn = -\frac{\mathrm{He}}{2\pi m} = n_1 - n,$$

$$-\frac{\mathrm{He}}{2\pi m} = n - n_2,$$

$$n_2 - n_1 = +\frac{\mathrm{He}}{\pi m},$$

which is the formula for the Zeeman effect except for the factor ½. This explanation of the Zeeman effect is open to a logical objection; namely, in Bohr's theory it is assumed that a line of the spectrum is emitted when an electron moves, we do not know on which path, from an outer to an inner non-radiating orbit. The magnetic field, of course, acts on the electron during the emission of light; that is, while the electron moves from one orbit to the other. But in this present theory, as in the older Lorentz theory, we assume that the magnetic field affects only the stationary orbits.

This assumption however remains valid for the determination of the diamagnetic susceptibility which we shall now consider.

$$dM = -\pi e a^2 \frac{dT}{T^2} = \frac{e^2 a^2 H}{2m}.$$

If there are N orbits per unit volume and if the axes are uniformly distributed in all directions, then we have

$$M = \frac{e^2 a^2 NH}{6M},$$

or, the diamagnetic susceptibility k is equal to

$$k = \frac{e^2 a^2 N}{6m},$$

or, twice as large as the same quantity calculated on Lorentz's assumption that the centripetal forces are proportional to the distance between the electron and the center of the atom. For the diamagnetic susceptibility and for the Zeeman effect we have to assume in Bohr's atom that under the influence of the magnetic field the nonradiating orbits are slightly changed.

For a paramagnetic gas the resultant susceptibility would be the difference

$$k = \frac{N_m M^2}{3RT} - \frac{e^2 a^2 N_a}{6m}.$$

For hydrogen we would have

$$k = \frac{4\pi^2 a^4 e^2 n^2 N}{3RT} - \frac{e^2 a^2 N 2}{6m}$$

$$= e^2 e^2 N \left[\frac{4\pi^2 a^2 n^2}{3RT} - \frac{2}{6m}\right].$$

If we take again for 0°

$$a = 5.22 \cdot 10^{-9},$$
$$n = 6.72 \cdot 10^{15},$$
$$m = 9.01 \cdot 10^{-28},$$
$$R = 1.37 \cdot 10^{-16},$$
$$T = 273.$$

We find:

$$\frac{4\pi^2 a^2 n^2}{3RT} = 4.31 \cdot 10^{29},$$
$$\frac{1}{3m} = 3.7 \cdot 10^{26},$$

that shows that the paramagnetic effect for hydrogen at 0° is more than 1,000 times larger than the diamagnetic effect.

For helium we have the corrected paramagnetic susceptibility equal to:

$$k = \frac{N_a M^2}{3RT} - \frac{e^2 a^2 N_a 2}{6m},$$
$$M = 2\pi a^2 en,$$
$$k = a^2 e^2 N_a \left[\frac{4\pi^2 a^2 n^2}{3RT} - \frac{1}{3m}\right].$$

With the previously given values for the radius a and the frequency n of the electrons we find

$$\frac{4\pi^2 a^2 n^2}{3RT} = 1.26 \cdot 10^{30}.$$

The relative diamagnetic effect of helium is a little smaller than the corresponding effect of hydrogen.

In general, the observed susceptibility k is the difference between the paramagnetic susceptibility k_p and the diamagnetic susceptibility k_d

$$k = k_p - k_d.$$

It is therefore quite conceivable that an element like tin changes at given temperatures from the negative to the positive sign of magnetism, and vice versa; k_p at all events is a function of temperature. For the diamagnetic gases the susceptibility is probably almost independent of temperature. If Bohr's theory of the structure of helium is in the right direction toward the physical reality then we have to make a little modification in order to explain the magnetic properties of helium. If the principle of conservation of the moment of momentum holds within

the atom, if this momentum is proportional to the magnet moment and if the electrons are moving in elliptic orbits, then the nucleus would also move around the center of attraction of the system, in the same direction as the electrons, the magnetic moments of the electrons and the nucleus would balance each other and the atom would be diamagnetic. Of course, it is not required that the resultant moment should be exactly zero, but only that the absolute value of k_d should be larger than k_p. If over a wide range of temperature of a gas k were independent of the temperature then k_p would be equal to zero. The Zeeman effect of iron vapor shows that the diamagnetic properties persist up to very high temperatures, and the additive law of diamagnetism for organic compounds of similar constitution indicates that the diamagnetism is a rather deep seated property of matter, which may be attributed partly to the nucleus. On the other hand the paramagnetic phenomena and the diamagnetic properties of solid and liquid substances are readily influenced by chemical and mechanical agencies.

Summary.

1. It has been shown that the relation

$$\frac{n_1}{n_2} = \frac{e_1}{e_2}$$

can be deduced by means of the quantum relation without Bohr's theory of the atom.

2. If we calculate the radii of the orbits and the velocities v of the electrons near the nucleus of the atom by means of the relations:

$$a_2 = a_1\frac{e_1}{e_2} = a_1\frac{N_1}{N_2},$$

$$v_2 = 2\pi n a_2,$$

we find for the greatest velocity of the electrons in the uranium atom $2.1 \cdot 10^{10}$ cm. corresponding to a mass m of the electron equal to $1.4m_0$.

3. The paramagnetic moments and the paramagnetic susceptibilities of hydrogen, helium and lithium have been calculated by means of Bohr's theory. If hydrogen really is diamagnetic, which has yet to be decided by experimental measurements, the magnetic properties of hydrogen and helium can not be explained by Bohr's atom-model. In this case a new model has to be invented, or the magnetic properties have to be ascribed to different causes, for instance, to proper magnetons, independent of revolving electrons, or to electrons whose charge itself is

in motion, so that the electron is at the same time a magneton or to the nucleus as being responsible partly for the magnetic phenomena.

4. A modification of the simple Zeeman effect has been given. The resultant equation

$$n_2 - n_1 = \frac{He}{\pi m}$$

differs from Lorentz's theory by the factor ½.

5. The diamagnetic susceptibility of hydrogen and helium has been calculated. It is about 1,000 times smaller than the paramagnetic susceptibility.

6. A modification of Bohr's atom has been considered, which will account for the diamagnetic properties of He.

Laboratory of Physics,
University of Illinois,
April, 1918.

THE MAGNETIC PROPERTIES OF SOME RARE EARTH OXIDES AS A FUNCTION OF THE TEMPERATURE.

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THE MAGNETIC PROPERTIES OF SOME RARE EARTH OXIDES AS A FUNCTION OF THE TEMPERATURE.

BY E. H. WILLIAMS.

THE existence or non-existence of the magneton, the elementary quantity of magnetism corresponding to the electron of electricity, has attracted the attention of investigators since it was first suggested by Ritz. Much work has been done, especially by Weiss, Kammerling Onnes, du Bois, Honda, Perrier, Piccard and Terry, and while the preponderance of the results obtained by these investigators is against the elementary moment of magnetism as suggested by Weiss, yet the idea is still strongly maintained by some. Within the last three years Piccard[1] maintains that there is a group of bodies (paramagnetic) which obey strictly Curie's law and hence he argues that the foundations of the theory are sound and there is still evidence in favor of the magneton.

Not only was it hoped to get evidence for or against the magneton theory, but the opportunity to obtain from the Chemistry Department of the University of Illinois rare earths in a very pure state from which were being made atomic weight determinations made it desirable to investigate their magnetic constants. My thanks are due to Dr. Hopkins and his associates of the Chemistry Department for their hearty cooperation in supplying me with samples of the rare earth materials.

The method used was that of Curie[2] which consists in measuring the pull exerted on an object placed in a non-uniform magnetic field. According to this method X, the magnetic susceptibility per unit mass, is given by the expression

$$X = \frac{1}{MH_y \frac{\partial H_y}{\partial x}} \cdot \frac{C\phi}{l} \qquad (1)$$

providing the force is measured by means of the twist of a suspension. In this expression M is the mass of the sample, H_y the field at right angles to the direction of motion of the sample, $\partial H_y/\partial_x$ the variation of this field along the direction of motion, C the couple necessary to twist the

[1] Piccard, Arch. des Sci. Phys. et Nat., 40, 278, 1915.

[2] P. Curie, Ann. de Chem. et de Phys., (7), 5, 298, 1895.

suspension through one radian, ϕ the angle of twist in radians and l the lever arm from the line of suspension to the sample.

The torsion balance was contained in a wooden box put together without the use of any magnetic material. The suspension consisted of a phosphor-bronze wire the torsion couple of which was 940 dyne cm. at 25° C. It was found that this varied with the temperature, the variation being about − 0.6 dyne cm. for 1° C. Since, in some cases, the box containing the torsion balance warmed up five or six degrees Centigrade, a correction was made for the variation of the torsion couple.

During the first part of the work an aluminum rod was used for the lever arm and pointer of the balance but the zero corrections for this were so large that it was deemed advisable to find another system. The system finally adopted consisted of glass which was slightly diamagnetic counterpoised with a small amount of aluminum which was slightly paramagnetic. By using the right amount of aluminum the zero correction could be made very small.

Several sets of values of H_y and dH_y/dx were plotted and from the mean curve values were taken for the product $H_y(dH_y/dx)$. These products were in turn plotted and the point in the field where the product was a maximum was determined. The apparatus was now set so that the center of the test coil was at the point in the field where the product $H_y(dH_y/dx)$ was a maximum and the values of H_y and dH_y/dx determined for various currents in the electromagnet. The mean of four such sets is given in Table I.

Table I.

$I.$	$H.$	$\Delta H/\Delta X.$	$H(\Delta H/\Delta X.)$
1.5 amp.	1020	205.4	209.5×10^3
2.0	1352	273.7	370.1
2.5	1687	338.5	571.1
3.0	2012	407.2	819.2
3.5	2351	473.0	1112.1
4.0	2659	530.7	1411.0
4.5	2968	596.4	1770.0
5.0	3267	648.9	2120.0

The couple C necessary to twist the suspension through unit angle was found by the ordinary vibration method. An accurately turned disc and ring were used for known moments of inertia and C calculated from the formula

$$C = \frac{4\pi^2 I}{(t'^2 - t^2)}, \tag{2}$$

where I is the known moment of inertia added to change the period from t to t'.

The oxides of seven of the rare earths, namely, erbium (Er_2O_3), dysprosium (Dy_2O_3), gadolinium (Gd_2O_3), samarium (Sa_2O_3), neodymium (Nd_2O_3), lanthanum (La_2O_3) and yttrium (Yt_2O_3), were investigated. Before using any of the oxides, they were heated to a temperature of about 1000° C. in a platinum crucible in order to decompose any carbonate which may have formed and to drive off all moisture.

The samples under investigation were contained in a silica capsule which was fitted to one arm of the torsion balance. The balance, with thermocouple and empty capsule in place, was first calibrated for fields due to currents of from 1.5 to 5 amperes and for temperatures from 25° C. to 300° C.

Table II. gives a sample set of data for Gd_2O_3 with the thermocouple

TABLE II.

Gd_2O_3 99 + *Per Cent. Pure.*

Mass of sample = .11912 gm. Lever arm of sample = 10.55 cm. Scale distance of torsion balance = 105.7 cm.

Temp.	Current in Magnet.	Deflection of Torsion Balance.	Corrected Deflection.	$X \times 10^6$.
21.8° C.	1.5 amp.	7.81 cm.	7.59 cm.	128.3
"	2.0	13.79	13.47	128.9
"	2.5	21.31	20.93	129.8
"	3.0	30.21	29.86	129.1
			Mean..........	129.0
103.2	1.5	6.12	5.98	101.1
"	2.0	11.11	10.93	104.6
"	2.5	16.80	16.64	103.2
"	3.0	23.74	23.61	102.1
"	3.5	31.82	31.75	101.1
			Mean..........	102.4
178.0	1.5	5.16	5.07	85.5
"	2.0	9.13	9.04	86.3
"	2.5	14.15	14.10	87.2
"	3.0	20.03	20.03	86.4
"	3.5	26.63	26.73	84.9
			Mean..........	86.1
269.5	1.5	4.15	4.07	68.6
"	2.0	7.43	7.37	70.3
"	2.5	11.65	11.63	71.9
"	3.0	16.49	16.59	71.5
"	3.5	21.87	22.10	70.2
			Mean..........	70.5

and torsion balance readings omitted. Four such sets of data were taken with different samples of the same material. One curve representing the four sets of data was drawn in which magnetic susceptibility was plotted against temperature. From this curve values were taken to test Curie's law.

In like manner the other oxides were tested and results plotted. From these curves the results in the first and third columns of Tables III., IV., V. and VI. were obtained. For each temperature the value of the magnetic susceptibility X, is the mean of the values obtained from four or five fields as in Table II.

Table III.

Gadolinium Oxide 99+ Per Cent. Pure.

t.	T.	$X \times 10^6$.	$XT \times 10^6$.	$X(T+12) \times 10^6$.
20	293	130.1	38119	39680
60	333	115.1	38328	39709
120	393	98.2	38593	39771
180	453	85.5	38731	39757
240	513	75.6	38783	39690
300	573	67.8	38849	39663

Table IV.

Erbium Oxide 99.6+ Per Cent. Pure.

t.	T.	$X \times 10^6$.	$XT \times 10^6$.	$X(T+13.5) \times 10^6$.
20	293	189.1	55406	57954
60	333	167.2	55678	57935
120	393	142.6	56042	57967
180	453	124.4	56354	58033
240	513	110.1	56462	57969
280	553	102.2	56516	57895

Table V.

Dysprosium Oxide 99.5+ Per Cent. Pure.

t.	T.	$X \times 10^6$.	$XT \times 10^6$.	$X(T+15) \times 10^6$.
20	293	234.1	68591	72103
60	333	207.4	69064	72175
120	393	176.7	69443	72094
180	453	153.9	69717	72025
240	513	136.6	70076	72125
300	573	122.6	70250	72089

TABLE VI.

Neodymium Oxide 99.5+ Per Cent. Pure.

t.	T.	$X \times 10^6$.	$XT \times 10^6$.	$X(T+44) \times 10^6$.
23	296	29.3	8672.8	9962.0
103.4	376.4	23.7	8920.7	9963.5
179.4	452.4	19.8	8957.5	9828.7
283.0	556	16.6	9229.6	9960.0

TABLE VII.

Samarium Oxide 99.5+ Per Cent. Pure.

Temp.	$X \times 10^6$.
22.3	6.02
101.8	5.93
270.2	5.98
Mean...........	5.98

TABLE VIII.

Lanthanum Oxide 99+ Per Cent. Pure.

Temp.	H_y.	Corrected Def.	$X \times 10^6$.
24° C.	2025	−0.09	−0.49
"	2660	−0.13	−0.41
"	3010	−0.14	−0.36
"	3328	−0.16	−0.34
		Mean..............	−0.40

TABLE IX.

Yttrium Oxide 99.5+ Per Cent. Pure.

Temp.	H_y.	Corrected Def.	$X \times 10^6$.
22° C.	2025	.09	.60
"	2351	.11	.52
"	2660	.13	.48
"	3010	.17	.51
"	3328	.21	.52
		Mean	.53

Samarium oxide, Table VII., shows no variation of magnetic susceptibility with temperature. Three sets of data similar to that in Table II. were taken, all of which are summarized in Table VII. In the case of lanthanum oxide and yttrium oxide, Tables VIII. and IX., the magnetic susceptibility was so small that no attempt was made to study the variation of the susceptibility with the temperature. In the case of lanthanum oxide the magnetic susceptibility is negative, thus indicating that this oxide is diamagnetic whereas all the others are paramagnetic.

According to Curie's law the susceptibility of paramagnetic bodies times the absolute temperature is equal to a constant; that is, $XT =$ constant. An examination of Tables III., IV., V. and VI. shows that the law does not hold for any of the materials investigated. However, they are found to follow quite closely a modification of Curie's law, namely, the susceptibility times the absolute temperature plus a constant is equal to a constant, $X(T+\theta) =$ constant, in which each material has its own value of θ.

TABLE X.

Oxideof	Williams. $X \times 10^6$	Levy. $X \times 10^6$
Yttrium	.53 (22° C.)	−.14
Lanthanum	−.40 (24° C.)	−.18
Neodymium	29.3 (23° C.)	33.5
Samarium	5.98	6.5
Gadolinium	130.1 (20° C.)	161.
Dysprosium	234.1 (20° C.)	290.
Erbium	189.1 (20° C.)	

Table X. gives a summary of the results obtained together with results quoted by Levy in his book on "The Rare Earths," page 153, 1915.

Various explanations have been advanced for the variation from Curie's law. Oosterhuis,[1] from a consideration of zero point energy, deduces an explanation as follows:

Taking the value of the rotational energy of the molecule as deduced by Einstein and Stern[2] to be

$$U = \frac{hn}{e^{hn/kT} - 1} + \tfrac{1}{2}hn,$$

where h is Planck's constant and n the frequency of the rotation, and further assuming that this rotational energy is inversely proportional to the magnetic susceptibility X as developed by Langevin,[3], Oosterhuis deduces the relation

$$X(T+\theta) = C, \quad \text{where} \quad \theta = \frac{1}{6}\cdot\frac{hn_0}{k}.$$

Since

$$n_0 = \frac{h}{4\pi^2 I},$$

where I is the moment of inertia of the molecule, he concludes that molecules with a small moment of inertia will have a large value of θ, a large zero point energy ($\tfrac{1}{2}hn_0$) and deviate markedly from Curie's law.

[1] Phys. Zeit., 14, 862, 1913.

[2] Ann. d. Phys., 40, 551, 1913.

[3] Ann. Chem. Phys., (8), 5, 70, 1905.

Although the results given above (Tables III., IV., V. and VI.) follow a modified form of Curie's law, θ does not vary inversely as the moment of inertia of the molecule as is shown in Table XI.

TABLE XI.

Oxide.	At. Wt.	θ.	Molecular Wt.	Molecular Wt. $\times \theta$.
Erbium...........	167.7	13.5	383.4	5176
Dysprosium........	162.5	15.	373.0	5595
Gadolinium........	157.3	12.	362.6	4351
Neodymium.......	144.3	44.	336.6	14810

Starting from an entirely different viewpoint Kunz[1] has derived the same expression for the variation of the magnetic susceptibility with the temperature. He points out that since it is quite likely that the electrons responsible for the paramagnetism revolve in the outer layer of the atom, the molecular moment will be the resultant of all the atomic moments of the atoms. Furthermore, with increasing temperature, it is quite likely that the atoms share the energy of temperature agitation which also will affect the resultant magnetic moment of the molecule. Therefore, in general, we may express the molecular moment as

$$M = M_0 f(T).$$

In solid or liquid paramagnetic substances the forces which oppose the tendency of the external field to direct the elementary magnets is composed of the temperature agitation RT together with a force due to the mutual effect of the molecules on each other and which would be a certain function of the temperature $f_1(T)$. With these assumptions Kunz obtains for particular values of $f(T)$ and $f_1(T)$,

$$X(T + \theta) = \text{constant}.$$

All of the substances included in this investigation which vary with the temperature obey the modified Curie law instead of the law itself. In the case of samarium oxide the magnetic susceptibility is found to be independent of the temperature. This is also probably true of lanthanum oxide and yttrium oxide.

It may appear from Table II. that the magnetic susceptibility varies with the field strength thus indicating that the substance is ferromagnetic in nature but the remainder of the data does not bear out this conclusion. In some cases the magnetic susceptibility came out very nearly constant for the different field strengths, while in others it varied in the opposite way to which it appears to vary in Table II. A careful study of all the data leads one to conclude that all the oxides are paramagnetic.

[1] PHYS. REV., VI., 2, 113, 1915.

It was thought worth while to test the accuracy of an analysis of rare earths by the magnetic method. To do this two pure substances were mixed in known proportions and the magnetic susceptibility of the mixture determined. The results, given in Tables XII. and XIII., show very close agreement between the percentage by weight and the percentage by the measurement of the magnetic susceptibility. The magnetic method would not be adaptable if the mixture consisted of more than two substances.

Table XII.

Er_2O_3	.09095 gm.
Yt_2O_3	.06735 gm.
Total	.15830 gm.
Per cent. of Er_2O_3 by weight	57.45 per cent.

Per Cent. by the Magnetic Method.

Substance.	$X \times 10^6$.
Er_2O_3 (99.6 per cent. pure)	187.7 (at 22.3° C.)
Yt_2O_3 (99.6 per cent. pure)	.53
Mixture	108.1 (at 22.3° C.)
Per cent. Er_2O_3 by the magnetic method	57.71 per cent.

Table XIII.

Per Cent. by Weight.

Er_2O_3	.06985 gm.
Sa_2O_3	.06767 gm.
Total	.13752 gm.
Per cent. of $Er_2\ O_3$ by weight	50.79 per cent.

Per Cent. by the Magnetic Method.

Substance.	$X \times 10^6$.
Er_2O_3 (99.6 per cent. pure)	187.9 (at 21.7° C.)
Sa_2O_3 (99.5 per cent. pure)	5.98
Mixture	98.9 (at 21.7° C.)
Per cent. of Er_2O_3 by magnetic method	51.08 per cent.

Summary.

The mass susceptibilities for the oxides of erbium, dysprosium, gadolinium, samarium, neodymium, lanthanum and yttrium have been measured for temperatures from 25° C. to 300° C. and all found to be paramagnetic with the exception of lanthanum which was slightly diamagnetic.

In those cases where the susceptibility varies with the temperature Curie's law is not found to hold but the results follow quite closely a modification of this law, namely, $X(T + \theta) = \text{constant}$. It follows that

in so far as Curie's law is essential to the existence or determination of the magneton the results obtained are unfavorable.

The magnetic susceptibility does not vary with the field strength.

The results are explainable either by the theory worked out by Kunz or the zero point energy theory of Oosterhuis.

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February, 1918.

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AMPLIFICATION OF THE PHOTOELECTRIC CURRENT BY MEANS OF THE AUDION.

BY CARL ELI PIKE.

A METHOD has been outlined by Jakob Kunz[1] by means of which the photoelectric current may be amplified, thus making the photoelectric cell more useful as a photometer, especially in the region of ultraviolet light and also for technical purposes.

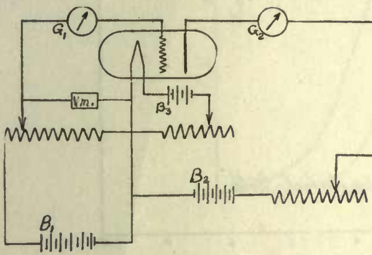

Fig. 1.

The amplification is produced by means of a vacuum tube with three electrodes, or the audion. In order to determine the best potentials to use in the primary and secondary circuits it is necessary to know the characteristic curves for the audions used. The characteristic curves of several audions have been determined by an arrangement of apparatus shown in Fig. 1, which is self explanatory. If we plot the grid potentials as abscissæ and the plate current as ordinates, the curve obtained is called the characteristic.

Due to the fact that the plate current as well as the grid current was so sensitive to small changes in the temperature of the filament, it was necessary to keep the heating current very constant. Large storage cells, well insulated from the ground, were used for this purpose. A large resistance was placed in the external circuit, so that a small variation in the resistance of the filament would not affect the current appreciably. The characteristic curves of three types of audions are shown in Figs. 3, 4 and 5. Audion no. 1 is an oscillion made by the DeForest Radio Telephone and Telegraph Co. Audion no. 2 of Fig. 4 is a W-type;

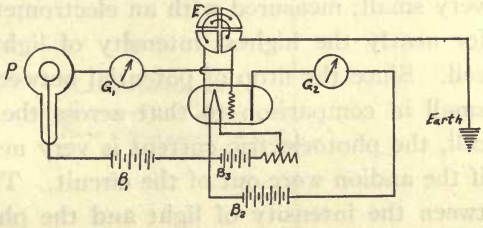

Fig. 2.

[1] PHYS. REV., Vol. X., No. 2, p. 205.

Audion no. 3 of Fig. 5 is a V-type instrument made by the Western Electric Co. It is noted that the plate current in the oscillion reaches its saturation value more abruptly than it does in either of the other two instruments. In the oscillion it is necessary to heat the filament to incandescence before the electrons are emitted, while in audion W and V the light from the filament was scarcely visible.

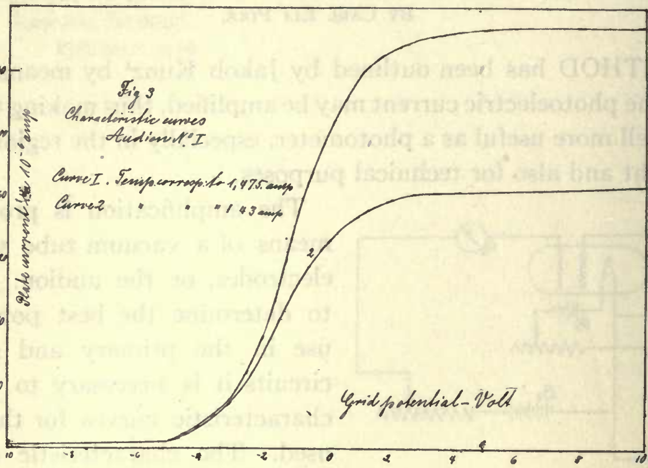

Fig. 3.

Audion no. 2, the W-type instrument, was used for the amplification of the photoelectric current, with an arrangement of apparatus shown in Fig. 2. Twenty-four volts were used in the secondary circuit and a hundred and twenty-five volts in the primary. The photoelectric cell used was a larger type of those made by Kunz in our laboratory. With 125 volts in the primary circuit, the drop of potential inside the audion was very small; measured with an electrometer it was found to be 0.56 volt for nearly the highest intensity of light incident on the photoelectric cell. Since the drop of potential between the grid and filament is very small in comparison to that across the terminals of the photoelectric cell, the photoelectric current is very nearly equal to what it would be if the audion were out of the circuit. The curve giving the relation between the intensity of light and the photoelectric current is shown in Fig. 6. It is unfortunately not a straight line. If this were a straight line and if the portion of the characteristic curve of the audion, used for the amplification, were straight, then we would expect a straight line relation between the intensity of light and the amplified current, and the amplification i_2/i_1, the ratio of the secondary to the primary current would be constant, represented by a straight line parallel to the hori-

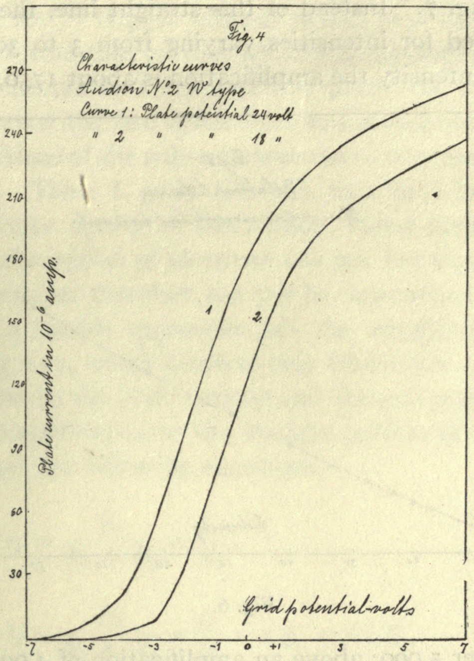
Fig. 4
Characteristic curves
Audion N°2 V type
Curve 1: Plate potential 24 volt
" 2 " " 18 "
Plate current in 10^{-6} amps.
Grid potential-volts
1
2
270
240
210
180
150
120
90
60
30
-7
-5
-3
-1
0
+1
3
5
7

Fig. 4.

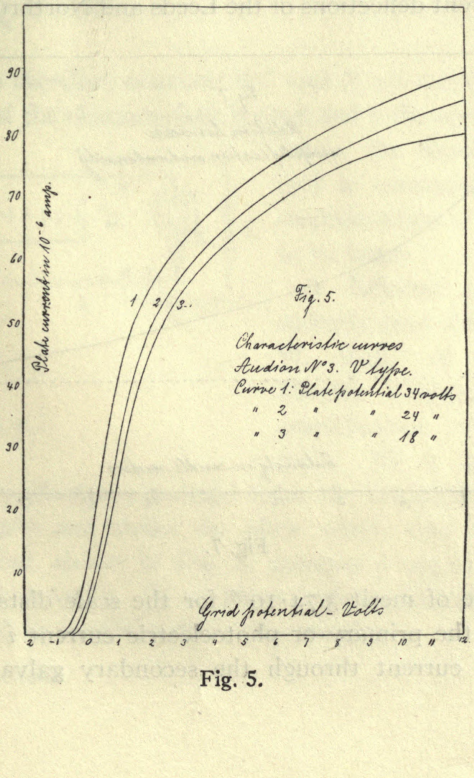
Fig. 5.
Characteristic curves
Audion N°3. V type.
Curve 1: Plate potential 34 volts
" 2 " " 24 "
" 3 " " 18 "
Plate current in 10^{-6} amp.
Grid potential- Volts
1
2
3
90
80
70
60
50
40
30
20
10
-2
-1
0
1
2
3
4
5
6
7
8
9
10
11
12

Fig. 5.

zontal axis of Fig. 7. Instead of this straight line, the curve of Fig. 7 has been obtained for intensities varying from 3 to 30 candle meters. For the highest intensity the amplification is about 1750, for the smallest

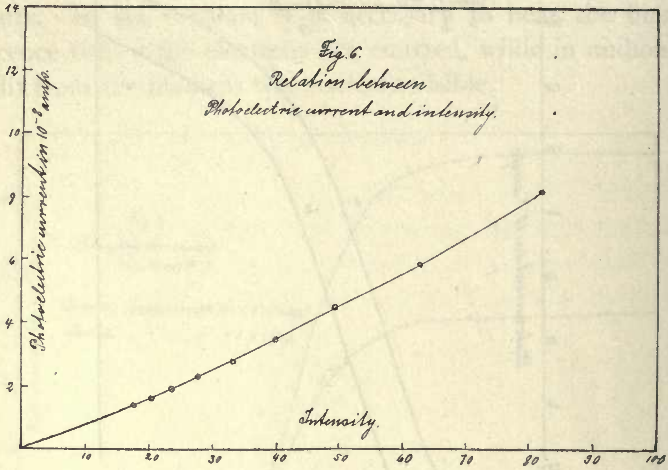

Fig. 6.

intensity it is over 5,000; above an amplification of 4,000 the points appear somewhat scattered around the curve, but this was only so because the primary current deflections of the Leeds and Northrup galvanometer,

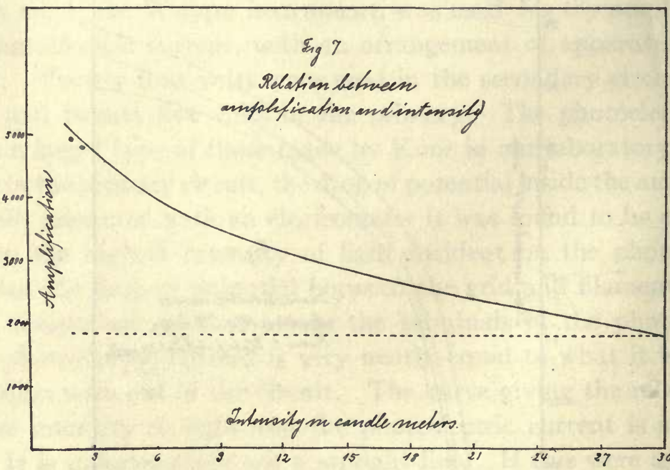

Fig. 7.

G_1 (with a figure of merit $3.74 \cdot 10^{-9}$ for the scale distance used), were very small. If the primary or photoelectric current i is zero, there is already a large current through the secondary galvanometer with a

figure of merit $2.9 \cdot 10^{-6}$. It goes without saying that this "dark" current was subtracted from that current which was obtained in the galvanometer, G_2, when the photoelectric cell was under the action of light. The difference between the two deflections was proportional to the current i_2. The deflections of the galvanometers were very steady and could easily be repeated. Table I. gives the data that have been plotted in Fig. 7. A satisfactory theory of the audion, based upon the motion, accumulation and absorption of electrons has not yet been given. The current amplification can therefore not yet be determined theoretically. But we can find a simple expression for the amplification, namely, i_2/i_1 in the following way, which involves only Ohm's law and the experimental relation between the plate current and the grid potential; as long as we restrict the amplification to the straight portion of the characteristic $i_2 = Cp_1$, we get the following equations.

$$i_1 = \frac{E_1}{R_0 + R_1} = \frac{p_1}{R_1},$$

$$i_2 = \frac{E_2}{R_2 + R_3} = \frac{p_2}{R_2} = Cp_1 = Ci_1R_1,$$

$$\frac{i_2}{i_1} = C \cdot R_1.$$

The amplification is therefore constant if C and R_1 are constants, that is, if the straight part of the characteristic is used and if the resistance R_1 between the filament and the grid is constant. For large amplifications C and R_1 have to be large.

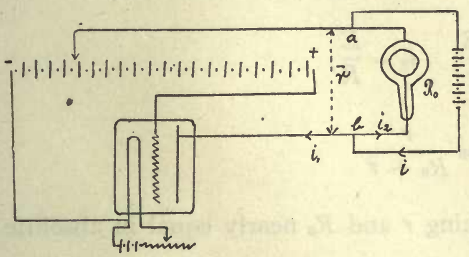

Fig. 8.

A different principle has recently been indicated by A. W. Hull,[1] of the General Electric Company, for the amplification of small currents. By a proper choice of the potentials, the electrons emitted from the incandescent filament pass through the grid and strike the plate where they are reflected. A system of this kind, shown in Fig. 8, presents a negative resistance $\bar{r}$ between the points a and b. If we place a photo-electric cell with the positive resistance R_0 in parallel with $\bar{r}$, then we get;

[1] P. I. R. E., February, 1918.

TABLE I.

Increase in d_2.	i_2 Amperes.	d_1.	i_1 Amperes.	Amplification.	Intensity in Candle Meters.
73.5	213.0×10^{-6}	35.0	131.0×10^{-9}	1600	33.0
64.8	188.0	23.5	87.7	2100	24.0
56.5	164.0	17.0	63.5	2600	18.5
49.0	142.0	12.9	48.2	2900	14.8
41.0	119.0	10.0	37.4	3270	12.0
35.5	101.5	8.1	30.2	3400	9.9
31.4	91.0	6.8	25.4	3570	8.3
28.0	81.2	5.7	21.3	3800	7.1
23.8	69.0	4.8	17.9	3840	6.1
22.5	65.2	4.1	15.3	4250	5.3
19.5	56.6	3.5	13.1	4320	4.7
17.5	50.7	3.1	11.6	4370	4.2
16.5	47.8	2.8	10.5	4570	3.7
15.1	43.7	2.5	9.4	4680	3.3
13.0	37.7	2.1	7.9	4800	3.0
12.0	34.8	1.9	7.1	4890	2.7
10.9	31.6	1.8	6.7	4720	2.5
11.5	33.3	1.7	6.4	5100	2.3
10.0	29.0	1.6	6.0	4840	2.1
10.0	29.0	1.5	5.6	5200	1.9

"W" type audion, no. 2. Lamp current, 3.75 amperes; candle power, 3.0. Heating current, 0.665 amperes. B^1 equaled 125 volts. B^2 equaled 25 volts. Figure of merit of G^1 3.74×10^{-9}. Figure of merit of G^2 2.9×10^{-6} when shunted with 1.7 ohm resistance. Ratio of figure of merit of G^2 to that of G^1 equaled 775.

$$i = i_1 + i_2 = E\left(\frac{R_0 + r}{Rr}\right),$$

and

$$i_1 = \frac{E}{r}, \quad i_2 = \frac{E}{R},$$

hence the amplification

$$\frac{i_2}{i} = \frac{r}{R_0 + r}$$

may be made very large by making r and R_0 nearly equal in absolute values.

One application of the amplification of photoelectric currents in wireless telegraphy may be pointed out. J. Kunz and J. Kemp[1] have at first used the photoelectric cell as receiver in wireless telegraphy. Their method has been modified by H. Behnken[2] who showed that the photoelectric cell with a string electrometer forms a constant detector of high sensitiveness, especially useful for photographic registrations. With

[1] Jahrbuch d. drahtlosen Telegraphie, 6, 405, 1913.

[2] Verhdlg d. deutschen phys. Ges., 16, p. 668, 1914.

the amplification of the weak currents here involved it should be possible to increase the usefulness of the photoelectric detector. It was intended to continue this investigation with ultra-violet and interrupted light, with alternating currents, and with a differential galvanometer in the secondary circuit.

Summary.

It has been shown that photoelectric currents can be amplified by means of the audion from 1,600 to 5,000 times. The weaker the light the smaller the primary photoelectric current and the larger the amplification. With different audions amplifications of 18,000 have been obtained.

In conclusion the writer wishes to express his appreciation to Jakob Kunz for suggesting the problem and directing the work.

Laboratory of Physics,
University of Illinois,
October, 1918.

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Summary.

It has been shown that photoelectric currents can be amplified by means of the audion from 1,000 to 5,000 times. The weaker the light the smaller the primary photoelectric current and the larger the amplification. With different audions amplifications of 18,000 have been obtained.

In conclusion the writer wishes to express his appreciation to Jakob Kunz for suggesting the problem and directing the work.

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